Newton's elemental law of shear stress – concepts of pressure

Question 1: Calculate the ratio of change in the volume of the water using E_{water}=19.62x10^4 N /cm^2 and Δp=100 atm and explain if the water could be considered as incompressible or not depending on your result. (E = Volumetric Elasticity Modulus)

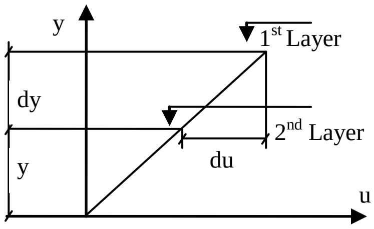
Solution 1:

$$1 \text{ atm} = 1 \text{ kg/cm}^2 \quad \Rightarrow \quad \Delta p = 100 \text{ kg/cm}^2$$
$$\Delta p = -E\ (\Delta V / V) \quad \Rightarrow \quad \textbf{Volumetric Change} = \Delta V / V = -\Delta p / \varepsilon = -5 \cdot 10^{-3}$$

The amount of volumetric change is very small and, so, it can be considered as incompressible.

Question 2: There is a 1.5 cm/s speed difference between two layers of a fluid, where the spacing between the two layers is 1 mm. The fluid is water and its kinematic viscosity is υ_{water} = 1·10^{-6} m^2/ s. Find the shear stress between the two layers in SI Unit system.

Answer: $\boldsymbol{\tau_{SI}}$ ***=0.015 N/m²***



Solution 2:

In the SI Unit System

$$\tau = \mu_{water} \frac{du}{dy} = \rho_{water} \cdot \upsilon_{water} \cdot \frac{du}{dy}$$
$$\upsilon_{water} = 10^{-6} \text{ m}^2/\text{s} \quad du = 1.5 \text{ cm/s} = 0.015 \text{ m/s} \quad dy = 1 \text{ mm} = 0.001 \text{ m}$$
$$\gamma_{water} = 9810 \text{ N m}^{-3} = \rho_{water} \cdot g \quad \Rightarrow \quad \rho_{water} = 1000 \text{ kg m}^{-3}$$
$$\tau = 1000 \times 1 \cdot 10^{-6} \times 0.015 / 0.001 = 0.015 \text{ N/m}^2 = 0.015 P_a$$

Question 3: At an unloading station, blocks weighing G=490.5 N are released from a smooth surface at an angle of 30° with a horizontal surface. Surface area of the blocks is A=0.2 m^2. The surface is greased with a pellicle having a thickness of 0.003 mm in order to get the blocks sliding with a constant downward speed of U=1.8 m/s. Find the velocity profile and dynamic viscosity of the pellicle (Thin oil layer between the blocks and the surface) . ***Answer: 2.04x10^{-3} Ns/m²***

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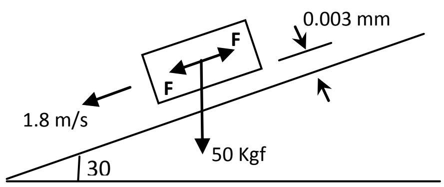
Solution 3:

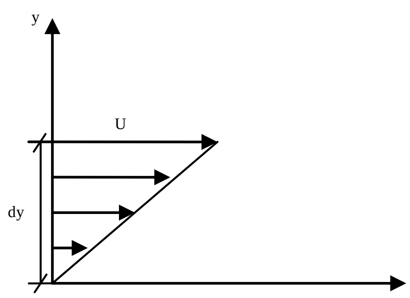


When the force (F_1) that moves the blocks downwards is equal to the viscous force (F_2) acting in the opposite direction, the acceleration will be zero and, thus, the velocity will be constant.

$$\Sigma\vec{F} = m \cdot a = \vec{F}_1 + \vec{F}_2$$
$$a = 0 \quad \Rightarrow \quad F_1 = -F_2 = P \cdot \sin 30^\circ = 245.25N$$
$$\tau = |F_2| / A = 1226.25 N/\mathrm{m}^2$$

On this kind of smooth surface, the fluid movement (greased oil) can be considered as parallel to the surface, which means that the flow is laminar. Because of the smallness in the thickness of the oil's layer, lets us assume that the velocity profile is linear.

$$\frac{du}{dy} = \frac{1.8-0}{0.003 \times 10^{-3}} = 6 \times 10^5 \, 1/\mathrm{s}$$
$$\tau = \mu_{\mathrm{oil}} \frac{du}{dy} \quad \Rightarrow \quad \mu_{\mathrm{oil}} = \frac{\tau}{du/dy} = 2.04 \times 10^{-3} \, N\mathrm{s/m}^2$$



Question 4: In a flowing fluid having a specific weight of 0.8 t (ton-force), the speeds of two layers that have 1 cm spacing between them are U_1=2 cm/s and U_2=3 cm/s, respectively. Find the shear stress in this region in terms of N/m^2**.** (γ_{oil} =0.8 t/ m^3; υ_{oil}=1·10^{-4} m^2/s) ***Answer: τ_{SI}=8x10^{-2}N/m^2***

Solution 4:

$$\gamma_{\mathrm{oil}} = 0.8\,\mathrm{t\,m^{-3}} = 7848\,N\,\mathrm{m}^{-3}$$
$$\gamma_{\mathrm{oil}} = \rho_{\mathrm{oil}} \cdot g \quad \Rightarrow \quad \rho_{\mathrm{oil}} = 800 \; N\mathrm{s}^2/\mathrm{m}^4$$
$$\tau = \mu_{\mathrm{oil}} \frac{du}{dy} = \rho_{\mathrm{oil}} \cdot \upsilon_{\mathrm{oil}} \cdot \frac{du}{dy}$$
$$\upsilon_{\mathrm{oil}} = 1 \cdot 10^{-4}\,\mathrm{m}^2/s \quad \mathrm{du} = u_1 - u_2 = 3 - 2 = 1\,\mathrm{cm/s} = 0.01\,\mathrm{m/s} \quad \mathrm{dy} = 1\,\mathrm{cm} = 0.01\,\mathrm{m}$$
$$\tau = 800 \times 1 \cdot 10^{-4} \times (1 \cdot 10^{-2}/1 \cdot 10^{-2}) = 8 \times 10^{-2} \;\; \mathrm{Nm}^{-2}$$

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Question 5: Given the absolute vapor pressure in a certain temperature of water is $p_{water,ab}$=0.23 t/ m^2, find the gage value of this pressure in terms of N /cm^2. (p_{atm}=9.81 N /cm^2) ***Answer: P_{gage}=-9.58 N/cm^2***

Solution 5:

Absolute pressure $p_{water,a} = 0.23\,\text{t}\,\text{m}^{-2} = 0.23 N/\text{cm}^2$ Gage pressure: $p_{water,g} = p_{water,a} - p_{atm} = -9.58\ \text{N}/\text{cm}^2$

Question 6: Assuming that the specific weight of the sea water is 1.02 t/m^3, find the absolute and gage pressure values at depth z=1000 m in terms of N /cm^2. (p_{atm}= 9.81 N/ cm^2) ***Answer: P_{gage}= 1000.62 N/cm^2, $P_{absolute}$ = 1010.43 N/cm^2***

Solution 6:

$$\gamma_{Sea} = 1.02\,\text{t}/\text{m}^3 = 10006.2 N/\text{m}^3 = 0.001001 N/\text{cm}^3 \qquad z = 1000\,\text{m} = 100000\,\text{cm}$$
$$p_{Gage} = \gamma_{Sea} \cdot z = 0.001001 \times 100000 = 1001 N/\text{cm}^2$$
$$p_{Absolute} = p_{Gage} + p_{atm} = \gamma_{Sea} \cdot z + p_{atm} = 1010.43 N/\text{cm}^2$$

Question 7: A diver is working in water at 25 m depth. How large is the pressure at this depth relative to the pressure at the surface of the water? (γ_{sea}=10055.25 N /m^3) ***Answer: $P_{25\ gage}$= 25.625 t/m^2, $P_{25\ absolute}$= 35.625 t/m^2***

Solution 7:

$$\gamma_{Sea} = 1.02\,\text{t}/\text{m}^3 = 10006.2 N/\text{m}^3 = 0.001001 N/\text{cm}^3 \qquad z = 1000\,\text{m} = 100000\,\text{cm}$$
$$p_{Gage} = \gamma_{Sea} \cdot z = 0.001001 \times 100000 = 1001 N/\text{cm}^2$$
$$p_{Absolute} = p_{Gage} + p_{atm} = \gamma_{Sea} \cdot z + p_{atm} = 1010.43 N/\text{cm}^2$$

Therefore, the pressure at the depth of 25 m, which is the excessive pressure the diver is exposed, is 25.63 t m^{-2} more than the pressure at the surface (p_{atm}).

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Question 8: A barometer reads h_1=74 cm at the foot of a mountain and it reads h_2=59 cm (mercury column) at the mountain peak. Find the height of the mountain.
Answer: ***$h_{mountain}$= 1606 m***

Solution 8:

$$p_{air,1} = \gamma_{mercury} \cdot h_1 = 13.6 \times 0.74 = 10.064 \text{ t/m}^2$$
$$p_{air,2} = \gamma_{mercury} \cdot h_2 = 13.6 \times 0.59 = 8.024 \text{ t/m}^2$$
$$\Delta p = p_{air,1} - p_{air,2} = 2.04 \text{ t/m}^2$$
$$1\text{t} = 9810\, N \quad \Rightarrow \quad \Delta p = 20012.4 N/\text{m}^2$$
$$\Delta p = \gamma_{air} \cdot H_{mountain} \Rightarrow \quad H_{mountain} = \Delta p / \gamma_{air} = 1606 \text{m}$$

Question 9: A cylinder with a mass m=1.962 N s^2/m is sliding downwards through a vertically positioned pipe. A thin oil layer exists between the cylinder and the pipe's internal surface. Axes of the cylinder and pipe overlap. (γ_{oil} =8044.2 N /m^3; υ_{oil}=$6 \cdot 10^{-6}$ m^2 /s)

a) Find the change in the speed of the cylinder in the pipe with respect to its unit displacement and the shear stress that acts upon the oil layer.
b) Find out the cylinder's terminal velocity inside the pipe. (Air pressure effect is neglected.)

Solution 9:

W: Cylinder's velocity and the velocity of the fluid on the point that the flow is contact with the cylinder.

The moment of the terminal velocity (W_f) of the cylinder is the time that it's weight (P) and viscous force (F) are in equilibrium.

$$\tau = \mu_{oil} \frac{dw}{dx} = \rho_{oil} \cdot \upsilon_{oil} \cdot \frac{dw}{dx}$$
$$\gamma_{oil} = \rho_{oil} \cdot g \quad \Rightarrow \quad \rho_{oil} = 820.61 \text{ Ns}^2/\text{m}^4$$
$$dw = W - 0 \qquad dx = 0.1\text{mm} = 0.0001\text{m}$$
$$dw/dx = (W/0.0001) 1/\text{s}$$
$$\tau = 820.61 \times 6 \cdot 10^{-6} \times W / 1 \cdot 10^{-4} = 49.24 \cdot W \; N/\text{m}^2$$