**Question 1:** For the reservoir pipe system given below:

**a-** Find the pressures of the fluid at points 1, 2 and 3.

**b-** Draw the energy and hydraulic grade lines.

**Solution 1:**

a) First calculate the discharge of the system.

Bernoulli's equation between A and B:

$$\frac{{V_A}^2}{2g}+\frac{P_A}{\gamma_{water}}+z_A=\frac{{V_B}^2}{2g}+\frac{P_B}{\gamma_{water}}+z_B \rightarrow 0+0+5=1+0+\frac{{V_B}^2}{2g} \rightarrow v_B=8.86m/s$$

$$Q=V_B A_S \rightarrow Q=8.86x\frac{\pi(0.1)^2}{4} \rightarrow Q=0.07m^3/s$$

From continuity equation:

$$Q=v_1 A_1 \rightarrow v_1=\frac{Q}{A_1} \rightarrow v_1=\frac{0.07}{\frac{\pi(0.3)^2}{4}} \rightarrow v_1=0.98\ m/s$$

$$v_2=\frac{Q}{A_2} \rightarrow v_2=\frac{0.07}{\frac{\pi(0.4)^2}{4}} \rightarrow v_2=0.56\ m/s$$

$$v_3=\frac{Q}{A_3} \rightarrow v_3=\frac{0.07}{\frac{\pi(0.2)^2}{4}} \rightarrow v_3=2.23\ m/s$$

If we write Bernoulli's equation between A and 1;

$$\frac{v_A}{2g}+\frac{P_A}{\gamma_{water}}+z_A=\frac{{v_1}^2}{2g}+\frac{P_1}{\gamma}+z_1 \rightarrow 5+0+0=2+\frac{P_1}{\gamma}+\frac{(0.98)^2}{19.62} \rightarrow \frac{P_1}{\gamma}=2.95\ m \rightarrow P_1 = 28.94kN/m^2$$

Between 1 and 2, Bernoulli's equation is written as:

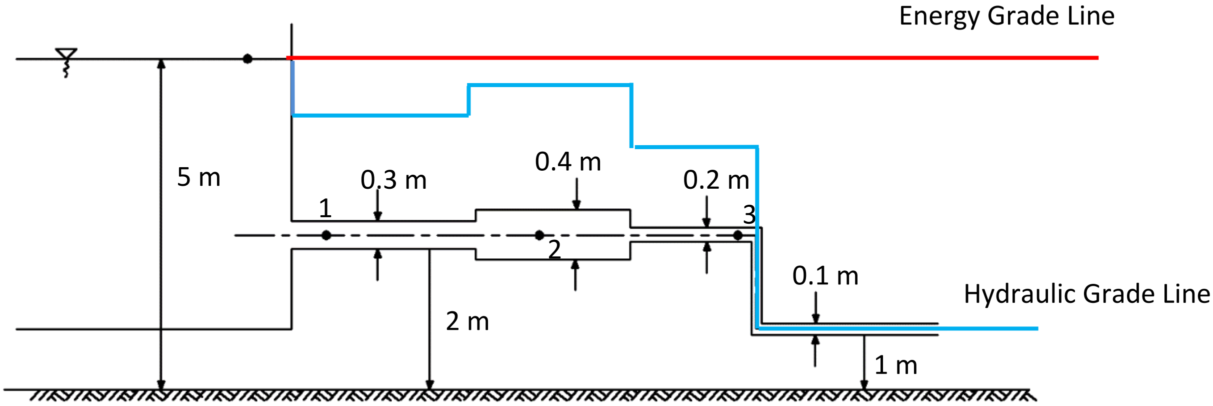
$$\frac{v_1}{2g}+\frac{P_1}{\gamma}+z_1=\frac{{v_2}^2}{2g}+\frac{P_2}{\gamma}+z_2 \rightarrow 2+28.94+\frac{(0.98)^2}{19.62}=2+\frac{P_2}{\gamma}+\frac{(0.56)^2}{19.62} \rightarrow \frac{P_2}{\gamma}=2.98\ m \rightarrow P_1=29.23kN/m^2$$

Between 2 and 3, Bernoulli's equation is written as:

$$\frac{v_2}{2g} + \frac{P_2}{\gamma} + z_2 = \frac{{v_3}^2}{2g} + \frac{P_3}{\gamma} + z_3 \rightarrow 2 + 29.23 + \frac{(0.56)^2}{19.62} = 2 + \frac{P_3}{\gamma} + \frac{(2.23)^2}{19.62} \rightarrow \frac{P_3}{\gamma} = 2.75\,m$$

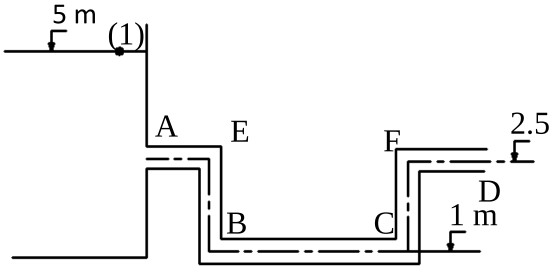
$$\rightarrow P_3 = 26.98 kN/m^2$$

b)




**Question 2:** The flow is opened to the atmosphere at point D which is fed by a very wide reservoir that has a water surface elevation of 5 meters as shown in the figure given below. Elevation of the horizontal axis BC is 1 meter. At sections AB and CD the diameter of the pipe is 0.2 meters. The flow is ideal (inviscid) and absolute atmosphere pressure is 9.81 N/cm$^2$.

- **a-** Considering the pipe's diameter is 0.15 m at section BC, find the discharges and the velocities of the flow at other sections. Draw the energy and hydraulic grade lines of the system.
- **b-** Assuming the system discharge is constant and has the value you have found in (a), calculate the minimum pipe diameter that the section BC can take and draw the energy and hydraulic grade lines of this situation. **Note:** Absolute vapor pressure of water is 2.26 kN/m$^2$.



**Solution 2:**

a- Since the fluid is ideal (inviscid) there is no friction and, therefore, there is no head loss. Thus, the system has only one quantity for discharge.

Bernoulli's equation between 1 and D:

$$\frac{{V_1}^2}{2g}+\frac{P_1}{\gamma_{water}}+z_1=\frac{{V_D}^2}{2g}+\frac{P_D}{\gamma}+z_D$$

$$0+0+5=\frac{{V_D}^2}{2g}+0+2.5\ \rightarrow V_D=7\,m/s\,;\ \frac{{V_D}^2}{2g}=2.5m$$

$$Q=v_DA_D\ \rightarrow Q=7x\frac{\pi(0.2)^2}{4}\ \rightarrow Q=0.22\,m^3/s$$

From continuity equation:

$$Q=v_{BC}A_{BC}\ \rightarrow\ v_{BC}=\frac{Q}{A_{BC}}=\frac{0.22}{\frac{\pi(0.15)^2}{4}}\ \rightarrow v_{BC}=12.45\,m/s\,;\ \frac{{v_{BC}}^2}{2g}=7.9m$$

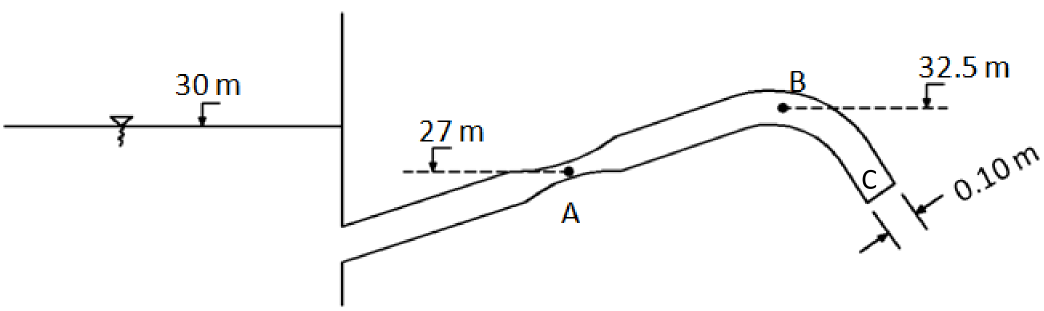
$$v_{CD}=v_{AB}=7\ m/s$$

b- Bernoulli's equation between 1 and BC;

$$\frac{{v_1}^2}{2g}+\frac{P_1}{\gamma}+z_1=\frac{{v_{BC}}^2}{2g}+\frac{P_{BC}}{\gamma}+z_{BC}\ \rightarrow 0+10+5=\frac{{v_{BC}}^2}{2g}+0.23+1\ \rightarrow v_{BC}=16.44\,m/s$$

$$Q=v_{BC}A_{BC}\ \rightarrow 0.22=16.44x\frac{\pi d_{BC}^2}{4}\ \rightarrow d_{BC}=0.13\ m;\ \frac{{v_{BC}}^2}{2g}=13.77m$$

**Question 3:** Discharge of the system shown in the figure given below is 50 lt/s. In case of an ideal fluid, what should be the dimeter of the pipe at point A in order for the pressures to be the same at points A and B?



***//Solution 3:**

Bernoulli's equation between A and B:

$$\frac{v_A^2}{2g}+\frac{P_A}{\gamma}+z_A=\frac{v_B^2}{2g}+\frac{P_B}{\gamma}+z_B \;\rightarrow\; P_A=P_B$$

$$\frac{v_A^2}{2g}+27=\frac{v_B^2}{2g}+32.5 \;\rightarrow\; \frac{v_A^2-v_B^2}{2g}=5.5m$$

$$50\,lt/s=0.05\,m^3/s$$

$$v_C=\frac{Q}{A_C}\;\rightarrow\;\frac{0.05}{\frac{\pi(0.1)^2}{4}}\rightarrow v_C=6.37\,m/s\rightarrow v_B=6.37\,m/s\,;\;\frac{v_B^2}{2g}=2.07m$$

$$\frac{v_A^2-(6.37)^2}{2g}=5.5m\;\rightarrow v_A=12.18\,m/s\rightarrow\frac{v_A^2}{2g}=7.56\,m$$

$$d_A^2=\frac{4}{\pi}\times\frac{0.05}{12.18}\;\rightarrow\; d_A=0.0723\text{ m}$$

The pipe acts like siphon.

Cross-section C is below the energy grade line.

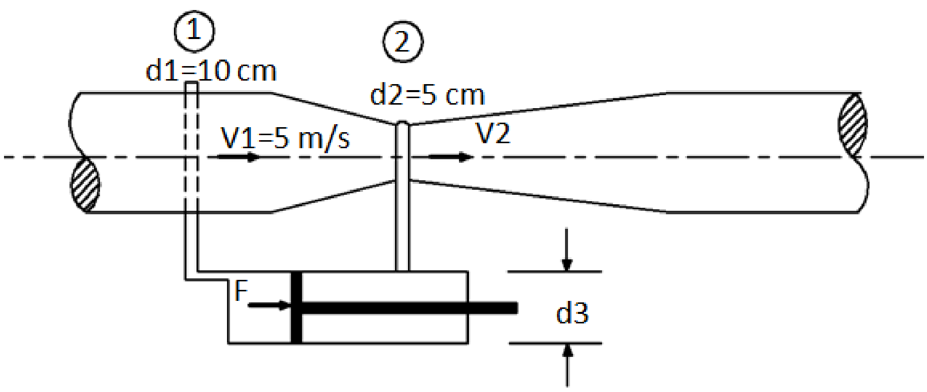
If it were above the energy grade line, it would not work.

If air is vacuumed from the part that is above, then it works.

**Question 4:** A venturimeter is mounted into a horizontal pipe as shown in the figure given below. A piston having a diameter, d3, of 3 cm is installed between sections "1" and "2".

a- Determine the pressure difference between sections "1" and "2", i.e. ΔP=P1-P2.

b- Compute the magnitude of the force, F, caused by the pressure difference and acting on the piston.



**Solution 4:**

**a)**

$$Q = v_1 \times A_1 \rightarrow Q = 5 \times \frac{\pi \times (0.1)^2}{4} \rightarrow Q = 0.04\, m^3/s$$

$$v_2 = \frac{Q}{A_2} = \frac{0.04}{\frac{\pi(0.05)^2}{4}} \rightarrow v_2 = 20.37\, m/s$$

Between 1-2, Bernoulli's equation is:

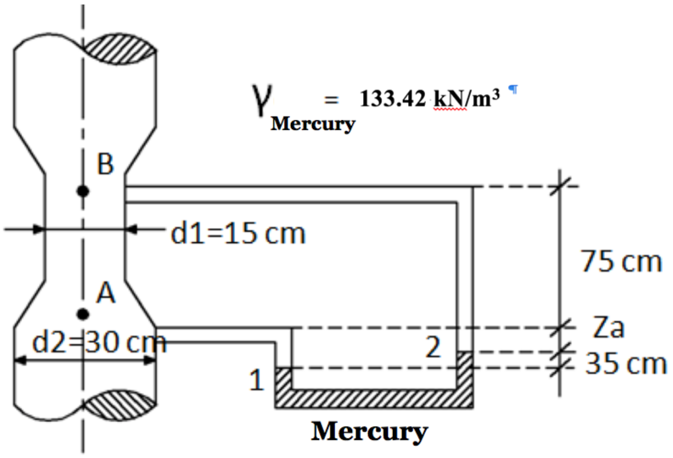
$$\frac{{v_1}^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{{v_2}^2}{2g} + \frac{P_2}{\gamma} + z_2 \rightarrow \frac{(5)^2}{2g} + \frac{P_1}{\gamma} + 0 = \frac{(20)^2}{2g} + \frac{P_2}{\gamma} + 0$$

$$\frac{P_1 - P_2}{\gamma} = 19.88\, m \rightarrow P_1 - P_2 = 195.02\, kN/m^2$$

**b)** $F = P \times A \rightarrow F = 19.88 \times 9.81 \times \frac{\pi \times (0.03)^2}{4} \rightarrow F = 0.14\, kN$

The venturi pipe which is shown in the figure is connected to the piston hose from above.

**Question 5:** A venturimeter is placed vertically as shown in the figure given below. Compute the discharge of water passing through the system by neglecting energy losses and by considering the indications of the manometer ($\gamma_{mercury}$=13.6 t/m³).




**Solution 5:**

$$P_{1left} = 0.35\gamma_{water} + z_A\gamma_{water} + P_A$$

$$P_{1right} = 0.35\gamma_{mercury} + z_A\gamma_{water} + 0.75\gamma_{water} + P_B$$

$$P_{1left} = P_{1right}$$

$$0.35\gamma_{water} + z_A\gamma_{water} + P_A = 0.35\gamma_{mercury} + z_A\gamma_{water} + 0.75\gamma_{water} + P_B$$

$$0.35x9.81 + z_A x9.81 + P_A = 0.35x133.42 + z_A x9.81 + 0.75x9.81 + P_B$$

$$P_A - P_B = 50.62\, kN/m^2 \rightarrow \frac{P_A - P_B}{\gamma} = 5.16m$$

From continuity equation:

$$Q = v_A A_A = v_B A_B \ \rightarrow v_A \frac{\pi(0.3)^2}{4} = v_B \frac{\pi(0.15)^2}{4}$$

$$v_B = 4v_A$$

Bernoulli's equation between A and B:

$$\frac{v_A^2}{2g} + \frac{P_A}{\gamma} + z_A = \frac{v_B^2}{2g} + \frac{P_B}{\gamma} + z_B$$

$$0 + \frac{P_A - P_B}{\gamma} = 0.75 + \frac{16v_A^2 - v_A^2}{2g}$$

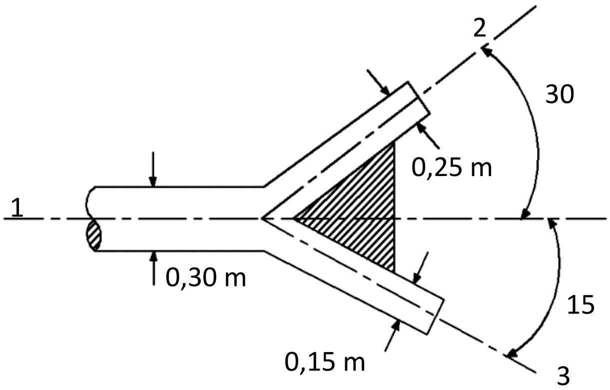
$$5.16 - 0.75 = \frac{15v_A^2}{2g} \rightarrow 4.41 = \frac{15v_A^2}{2g}$$

$$v_A = 2.40\, m/s$$

$$v_B = 9.60\, m/s$$

$$Q = 2.40 \frac{\pi(0.3)^2}{4} \rightarrow Q \ \cong 0.17\, m^3/s \ or \ Q \cong 170\ lt/s$$

**Question 6:** The water flow in a horizontal pipe is split into two equal-velocity flows as shown in the figure given below. The flow velocity in pipes "2" and "3" of the system is 2.5 m/s. Compute the discharges of water in pipes 2 and 3 and compute the magnitude of the force exerted on the section where the flow is split into two parts (End of the pipes "2" and "3" are open to the atmosphere).




**Solution 6:**

$$v_2 = v_3 = 2.5\, m/s$$

$$Q_2 = v_2 A_2 \ \rightarrow Q = 2.5 \times \frac{\pi(0.25)^2}{4} \ \rightarrow Q = 0.123\, m^3/s$$

$$Q_3 = v_3 A_3 \ \rightarrow Q = 2.5 \times \frac{\pi(0.15)^2}{4} \ \rightarrow Q = 0.044\, m^3/s$$

$$Q_1 = Q_2 + Q_3 \ \rightarrow \ Q_1 = 0.167\, m^3/s$$

$$v_1 = \frac{Q_1}{A_1} \rightarrow v_1 = \frac{0.167}{\frac{\pi(0.8)^2}{4}} \rightarrow v_1 = 2.36\,m/s$$

Bernoulli's equation between 1 and 2:

$$\frac{{v_1}^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{{v_2}^2}{2g} + \frac{P_2}{\gamma} + z_2 \rightarrow \frac{(2.36)^2}{2g} + \frac{P_1}{\gamma} + 0 = \frac{(2.5)^2}{2g} + \frac{P_2}{\gamma} + 0 \rightarrow P_1 = 0.29\,kN/m^2$$

$$\rho\left(\varrho_ç\vec{v}_ç - \varrho_g\vec{v}_g\right) = \sum \vec{F} = B\vec{K} + K\vec{K} + \vec{R}$$

x - direction:

$$\sum \vec{F} = B\vec{K} + K\vec{K} + \vec{R} = \rho(\varrho_2\vec{v}_2\cos 30 + \varrho_3\vec{v}_3\cos 15 - \varrho_1\vec{v}_1) = P_1A_1 + R_x$$

$$\frac{1}{9.81}\left(0.123 \times 2.5 \times \frac{\sqrt{3}}{2} + 0.044 \times 2.5 \times 0.966 - 0.167 \times 2.36\right) = 0.29 \times \frac{\pi(0.3)^2}{4} + R_x$$

$R_x = -0.04\,kN$ (Force acting on the flow by elbow.)

$R'_x = 0.04\,kN$ (Force acting on the elbow by flow.)

y - direction:

$$\rho(\varrho_2\vec{v}_2\sin 30 + \varrho_3\vec{v}_3\sin 15) = R_y$$

$R_y = 0.13\,kN$

$R'_y = -0.13\,kN$

$$R' = \sqrt{(R_x)^2 + \left(R_y\right)^2} \rightarrow R' = 0.14kN$$

**Question 7:** Schematic view of a hydroelectric power plant is given in the figure below. Discharge of the system where water is carried from the reservoir to the power station via a gallery, surge tank, and a pressurized pipe is designed to be 10 $m^3$/s. The weight of water between the sections "1" and "2" is determined to be 250 kgf during the run of the system. Compute the minimum block weight which should be mounted between these two sections ($z_1$=10m and $z_2$=8 m; $D_1$=1.6 m and $D_2$=1.5 m; $p_1$=140 kN/$m^2$).

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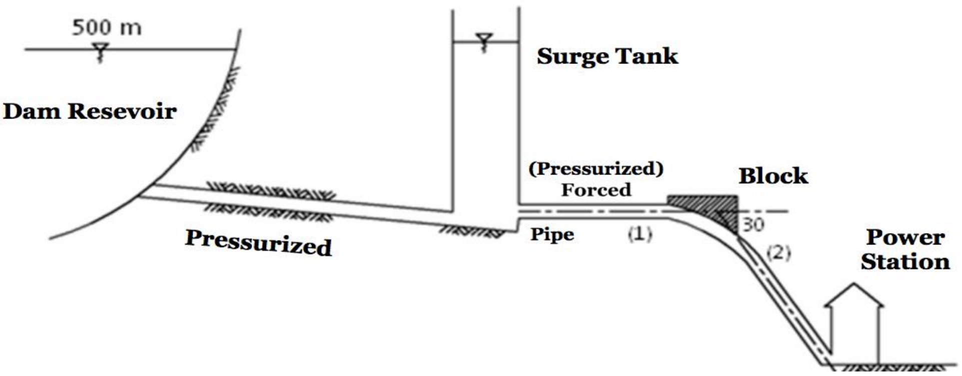
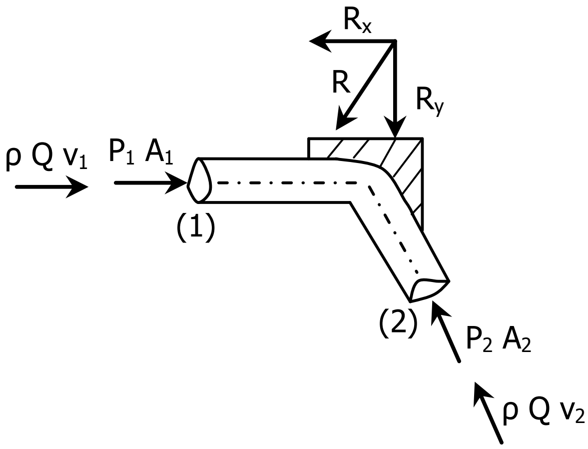


**Solution 7:**




$$v_1 = \frac{Q}{A_2} \rightarrow v_1 = \frac{10}{\frac{\pi(1.6)^2}{4}} \rightarrow v_1 = 4.97\ m/s$$

$$Q = v_1 \times A_1 = 5 \times \frac{\pi(1.6)^2}{4} \rightarrow Q = 10\ m^3/s$$

$$v_2 = \frac{Q}{A_2} \rightarrow v_2 = \frac{10}{\frac{\pi(1.6)^2}{4}} \rightarrow v_2 = 4.97\ m/s$$

Bernoulli's equation between 1 and 2:

$$\frac{{v_1}^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{{v_2}^2}{2g} + \frac{P_2}{\gamma} + z_2 \rightarrow \frac{(4.97)^2}{2g} + 14 + 10 = \frac{(4.97)^2}{2g} + \frac{P_2}{\gamma} + 8 \rightarrow \frac{P_2}{\gamma} = 16\ m \rightarrow P_2 = 160\ \ kN/m^2$$

Since the weight of the block is asked, we should examine the forces in the y direction.

$$-\rho\varrho v_2 \sin 30 = P_2 A_2 \sin 30 - w_{su} + R_y$$

$$\frac{1}{9.81} \times 10 \left(-4.97 \times \frac{1}{2}\right) = 160 \times \frac{\pi(1.6)^2}{4} \times \frac{1}{2} - 2.45 + R_y \rightarrow R_y = -160.85\ kN$$

This means that the force acts on the flow in the opposite direction. Since the block is considered as an external force in our case, for the equilibrium to be obtained, the force of the flow acting on the elbow should be the same with external force.

$$R'_y = 160.85\ \ kN$$

Let's examine the forces in the x direction.

$$\rho Q v_1 + P_1 A_1 - R_x - P_2 A_1 \cos 30 - \rho Q v_2 \cos 30\ = 0$$

$$\rho Q (v_1 - v_2 \cos 30) + P_1 A_1 - P_2 A_1 \cos 30 - R_x = 0$$

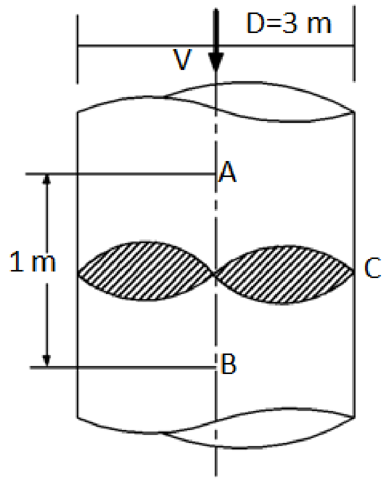
$$\frac{1}{9.81} \times 10 \times (4.97 - 4.97 \times 0.866) + 140 \times \frac{3.14 \times (1.6)^2}{4} - 160 \times \frac{3.14 \times (1.6)^2}{4} \times 0.866 = R_x$$

$$0.678 + 281.64 - 278.45 = R_x \rightarrow R_x = 3.57\ kN$$

$$R = \sqrt{(3.57)^2 + (160.85)^2} \rightarrow R = 160.89\ kN$$

**Question 8:** The velocity of the flow in the pipe shown in the figure given below is measured as 3 m/s at section "A". Compute the power transmitted to the turbine propeller (mounted at section "C") by taking into consideration the relative pressure values at sections "A" and "B" of the pipe as 10 kgf/cm² and -0.1 kgf/cm², respectively. The pipe has a uniform diameter and the fluid is water.

**Solution 8:**




$N = Q \times H_{turb} \times \gamma_{water}$

$Q = v_A \times A_A \rightarrow Q = 3 \times \frac{\pi \times (3)^2}{4} \rightarrow$

$Q = 21.206\, m^3/s\ (v_A = v_B)$

$H_A = H_B$ If the turbine did not exist,

$H_A - H_B = H_{turbine}$

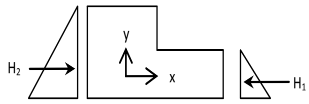
$\frac{v_A^2}{2g} + \frac{P_A}{\gamma} + z_A = \frac{v_B^2}{2g} + \frac{P_B}{\gamma} + z_B + H_{turbine}$

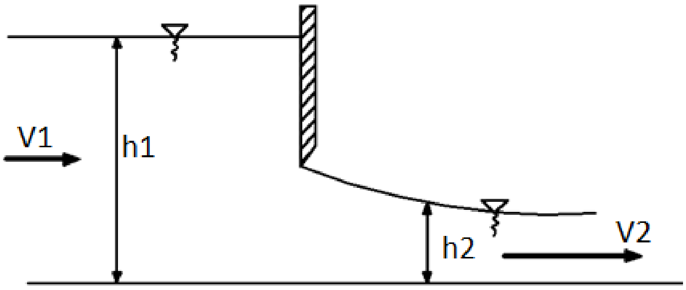
$100 + 1 = -1 + 0 + H_{turbine} \rightarrow H_{turbine} = 102\ m$

$N = 21.206 \times 102 \times 9810 \rightarrow N = 21.22 \times 10^6\ N.m/s = 28840\ BB$

**Question 9:** There is an incompressible and uniform flow between cross sections (1) and (2) of the sluice way shown in the figure given below. Considering the pressure variation is hydrostatic and the streamlines are parallel to each other on the cross-sections, find the direction and the magnitude of the force that the flow exerts on the sluice way. (Width of the sluice gate is b).

**Solution 9:**




Pressure forces;

$H_1 = \gamma \times \frac{h_1^2}{2} \times b\ ve\ H_2 = \gamma \times \frac{h_2^2}{2} \times b$

$Q = v_1 A_1 = v_2 A_2 \rightarrow v_1 h_1 b = v_2 h_2 b \rightarrow v_1 h_1 = v_2 h_2$

$\sum \vec{F} = B\vec{K} + K\vec{K} + \vec{R} = \rho(\rho_{out}\vec{v}_{out} - \rho_{in}\vec{v}_{in})$

$\rho Q(v_2 - v_1) = P_1 A_1 - P_2 A_2 + 0 + R_x$

$\rho Q\left(v_1 \frac{h_1}{h_2} - v_1\right) = \gamma \frac{h_1^2}{2} b - \gamma \frac{h_2^2}{2} b + R_x$

$\rho Q v_1 \left(\frac{h_1}{h_2} - 1\right) = \frac{\gamma b}{2}(h_1^2 - h_2^2) + R_x$

$$\rho v_1 h_1 b v_1 \left(\frac{h_1}{h_2} - 1\right) = \frac{\gamma b}{2}(h_1^2 - h_2^2) + R_x$$

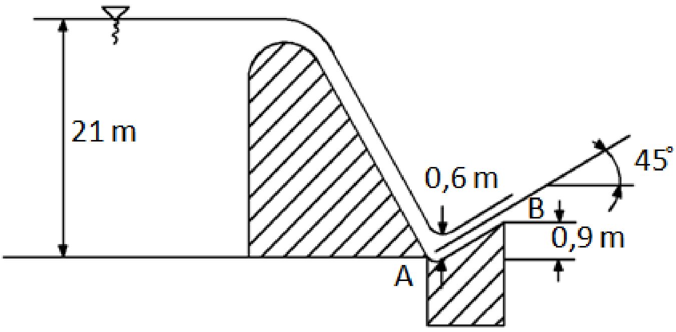
$\rho h_1 b {v_1}^2 \left(\frac{h_1}{h_2} - 1\right) - \frac{\gamma b}{2}(h_1^2 - h_2^2) = R_x$ The force that the sluice way exerts on the flow

$R'_x = \frac{\gamma b}{2}(h_1^2 - h_2^2) - \rho h_1 b {v_1}^2 \left(\frac{h_1}{h_2} - 1\right)$ The force that the flow exerts on the sluice way

**Question 10:** For the system given below:

a- Calculate the velocities and the discharges at cross-sections A and B.

b- Find the horizontal and vertical components of the force acting on the shaded obstacle. (Water weight between the A-B cross-sections is 2.65 kN.)



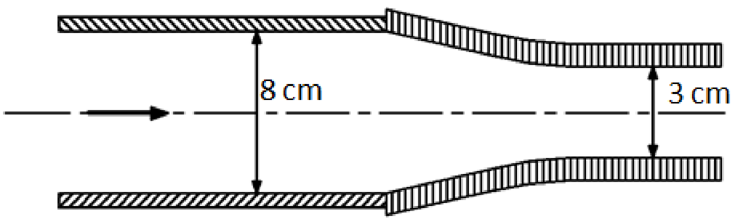

Results: $v_a = v_b = 20\, m/s\, ; q = 12\, m^3/s\, /m$ b- $R = 186.78\, kN$

**Question 11:** The inner diameter of a nozzle, which is attached to an 8 cm diameter garden hose is 3 cm.

a. Compute the energy at the end section of the nozzle, i.e. at the point where the water leaves the system, for a given discharge of 50 l/s.

b. Compute the forces acting on the nozzle for both of the following cases:

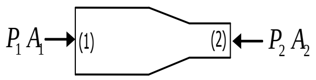
1. The system is running and its discharge is equal to 50 l/s.
2. The system is not running and the water is at rest in the garden hose.



**Solution 11:**

**a)**

$H = \frac{v^2}{2g} + \frac{P}{\gamma} + z$




$Q = 50\, lt/s = 0.05 m^3/s$

$$v_1 = \frac{Q}{A_1} \rightarrow v_1 = \frac{0.05}{\frac{\pi x (0.08)^2}{4}} \rightarrow v_1 = 9.95\, m/s \;\; and \;\; v_2 = 70.74\, m/s$$

Bernoulli's equation between 1 and 2;

$$\frac{{v_1}^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{{v_2}^2}{2g} + \frac{P_2}{\gamma} + z_2$$

$$\frac{(9.95)^2}{2g} + \frac{P_1}{\gamma} + 0 = \frac{(70.74)^2}{2g} + 0 + 0 \rightarrow P_2 = 2452.5\, kN/m^2$$

The head the fire hose works at $= H = \frac{{v_2}^2}{2g} + \frac{P_2}{\gamma} + z_2 \rightarrow H = \frac{(70.74)^2}{2g} + 0 + 0 \rightarrow H = 255\, m$

b-1. When the nozzle of the hose is open:

$$\rho Q(\overrightarrow{v_ç} - \overrightarrow{v_g}) = \sum F \rightarrow \rho Q(v_2 - v_1) = P_1 A_1 + R_X$$

$$\frac{1}{9.81} x 0.05 (70.74 - 9.95) = 2452.5 x \frac{\pi (0.08)^2}{4} + R_x$$

$$R_x = -9.32\, kN \rightarrow R'_x = 9.32\, kN$$

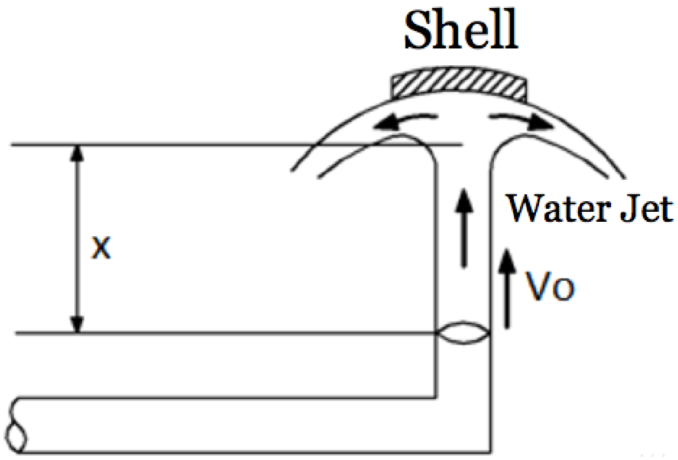
b-2. When the nozzle is closed:

Velocity = 0 → $P_1 = P_2$

$$\rho Q(v_2 - v_1) = P_1 A_1 - P_2 A_2 + R_X \rightarrow 0 = \frac{2452.5 x \pi}{4}[(0.08)^2 - (0.03)^2] + R_X$$

$$R_x = -10.1\, kN \rightarrow R'_x = 10.1\, kN$$

**Question 12:** A water jet that comes out of a pipe holds the hemispheric Shell which weighs 490.5 N as depicted in the figure given below. Neglecting the air resistance and the water jet's weight, find the jet's elevation in this situation. (Exit velocity Vo=10 m/s, exit cross-sectional area A=0.005 $m^2$).




**Solution 12:**

$Q = v_0 A_0 \rightarrow Q = 10x0.005 \ \rightarrow Q = 0.05\, m^3/s$

$Q = Q_1 + Q_2$

Cross-section areas are constant and equal to each other.

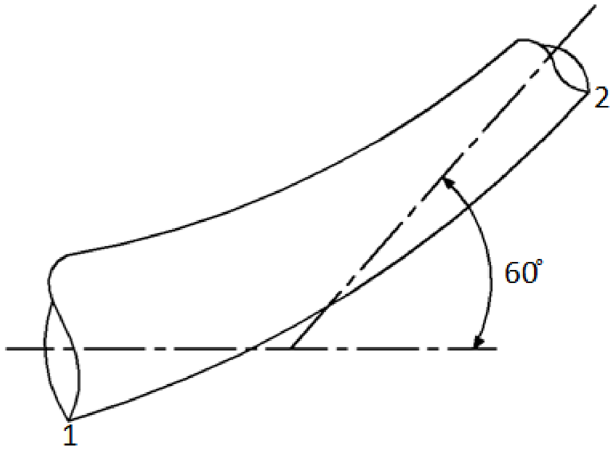
If we write the momentum equation,

$$R_y = \rho Q(\overrightarrow{v_{out}} - \overrightarrow{v_{in}}) \ \rightarrow 0.4905 = \frac{10}{9.81}\left(\frac{v}{2} + \frac{v}{2} + v\right) \ \rightarrow v = 4.90\, m/s$$

Bernoulli's equation between 0 and 2;

$$\frac{{v_0}^2}{2g} + \frac{P_0}{\gamma} + z_0 = \frac{{v_2}^2}{2g} + \frac{P_2}{\gamma} + z_2 \ \rightarrow \ \frac{(10)^2}{2g} + 0 + 0 = \frac{(4.9)^2}{2g} + 0 + x \ \rightarrow x = 3.87\, m$$

**Question 13:** Find the magnitude and the direction of the resultant force that acts upon the elbow which is located on the horizontal plane. Diameter of pipe at section (1) is $D_1$=600 mm, pressure is $p_1$=166.77 kN/m$^2$, the diameter at section (2) is $D_2$= 300 mm and the discharge is Q=1 m$^3$/s.




**Solution 13:**

$$\sum \vec{F} = B\vec{K} + K\vec{K} + \vec{R} = \rho(Q_{out}\vec{v}_{out} - Q_{in}\vec{v}_{in})$$

$$v_1 = \frac{Q}{A_1} \rightarrow v_1 = \frac{1}{\frac{\pi \times (0.6)^2}{4}} = 3.54\, m/s \rightarrow$$

$$v_2 = \frac{Q}{A_2} = \frac{1}{\frac{\pi \times (0.3)^2}{4}} = 14.15\, m/s$$

Bernoulli's equation between 1 and 2:

$$\frac{{v_1}^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{{v_2}^2}{2g} + \frac{P_2}{\gamma} + z_2 \rightarrow \frac{(3.54)^2}{2g} + 17 + 0 = \frac{(14.15)^2}{2g} + \frac{P_2}{\gamma} + 0 \rightarrow \frac{P_2}{\gamma} = 7.44\, m$$

$$\rightarrow P_2 = 72.99\ \ kN/m^2$$

**For x-direction:**

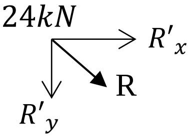
$$\frac{\gamma}{g}(Qv_2 \cos 60 - Qv_1) = P_1A_1 + P_2A_2 \cos 60 + R_x$$

$$\frac{1}{9.81}\left(14.15\frac{1}{2} - 3.54\right) = 166.77 \times \frac{\pi(0.6)^2}{4} + 72.99 \times \frac{\pi(0.3)^2}{4} \times \frac{1}{2} + R_x \rightarrow R_x = -49.73\, kN$$

The force that the pipe section exerts on the flow, $R'_x = 49.73\, kN$

**For y-direction:**

$$R = \sqrt{(R'_x)^2 + \left(R'_y\right)^2} \rightarrow R = \sqrt{(4.51)^2 + (1.70)^2} \rightarrow R = 44.24kN$$




$$\frac{\gamma}{g}(Qv_2 \sin 60 - 0) = -P_2A_2 \sin 60 + R_Y$$

$$\frac{1}{9.81}\left(1 \times 14.15 \times \frac{\sqrt{3}}{2} - 3.54\right) = -72.99 \times \frac{\pi(0.3)^2}{4} \times \frac{\sqrt{3}}{2} + R_y \rightarrow R_y = 16.68\, kN$$

$$R = \sqrt{(R'_x)^2 + \left(R_y\right)^2} \rightarrow R = 44.24\, kN$$