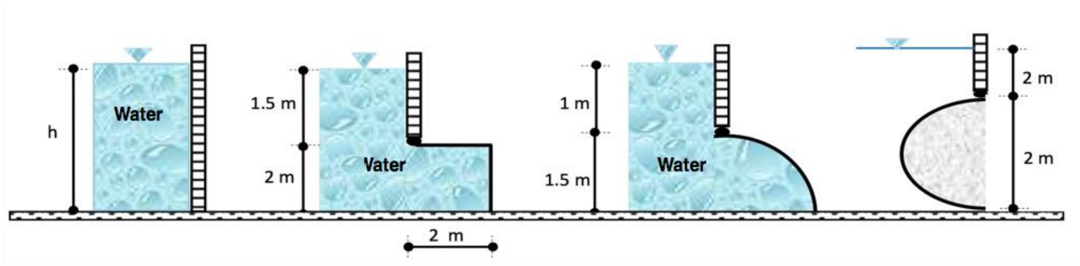
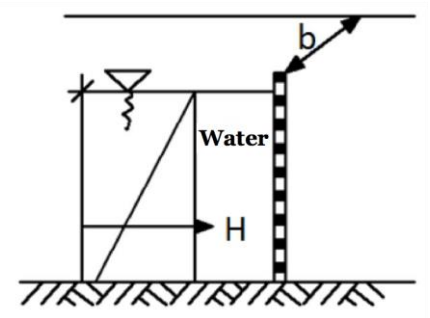
Pressure Forces

Question 1: Find the horizontal and vertical forces acting below the point shown in the figure that the gate is hinged. (Width perpendicular to the figure plane is 1m).

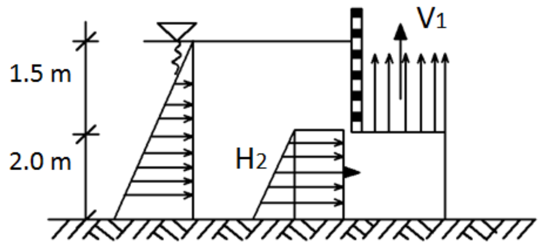


Solution 1:



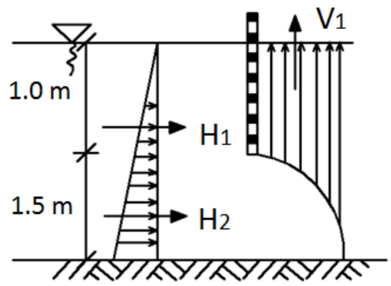
$$P_{atm} = 0$$

$$H = \frac{1}{2} x \gamma_{water} x h^2$$



$$H_2 = \frac{1}{2} x \gamma_{water} x [(3.5)^2 - (1.5)^2] = 49.05kN$$

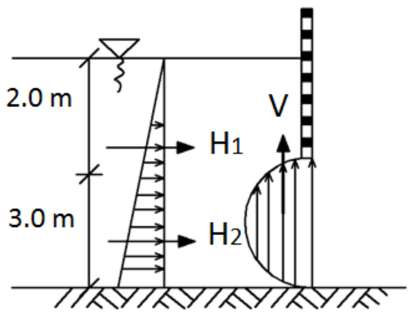
$$V_1 = \gamma_{water} x 1.5 x 2 = 29.43kN$$



$$H_1 = \frac{1}{2} x \gamma_{water} x (1)^1 = 4.91kN$$

$$H_2 = \frac{1}{2} x \gamma_{water} x [(2.5)^2 - (1)^2] = 25.70kN$$

$$V_1 = \left[2.5x1.5x1x10 - \frac{\pi x 3^2}{16} x1x10 \right] = 19{,}82kN$$



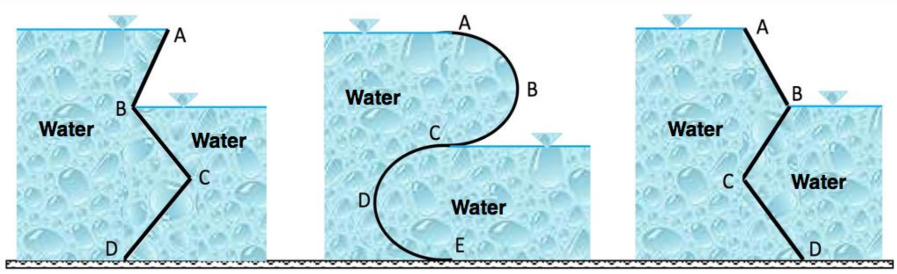
$$H_1 = \frac{1}{2} x \gamma_{water} x (2)^2 = 19.62kN$$

$$H_2 = \frac{1}{2} x \gamma_{water} x [(5)^2 - (2)^2] = 103.01kN$$

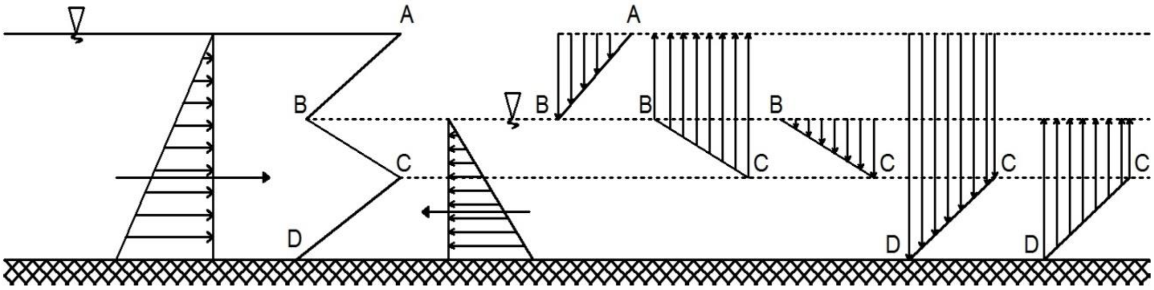
$$V_1 = \gamma_{water} x \frac{1}{2} \left(\frac{\pi x 3^2}{4} \right) = 34.73kN$$

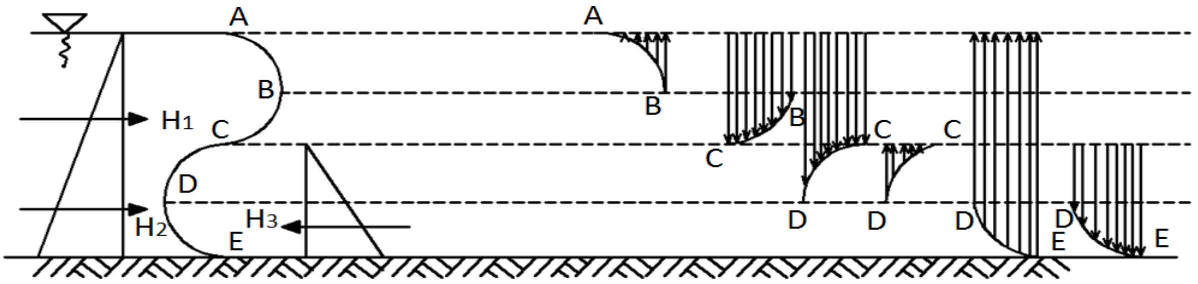
Pressure Forces

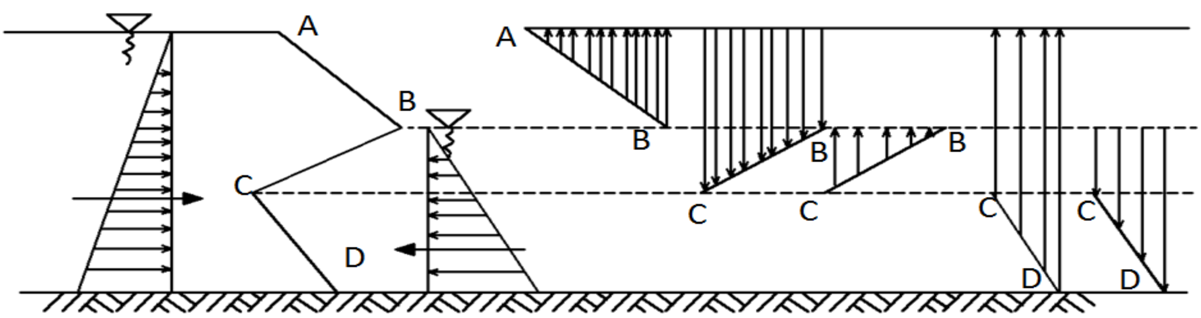
Question 2: Schematize the horizontal and vertical pressure forces acting on the surfaces ABCD. (Width perpendicular to the figure plane is 1m).



Solution 2:







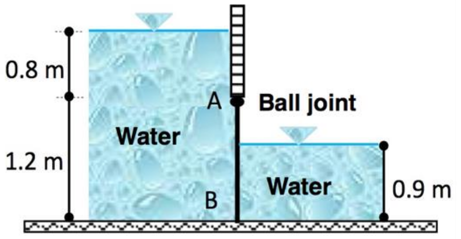
Question 3: The width of gate AB that is hinged at point A is 2 meters and the gate is part of the wall separating the chamber into two parts.

a. Find the direction and the magnitude of the force that should be applied to point B to hold the gate in position if there is water inside the parts and

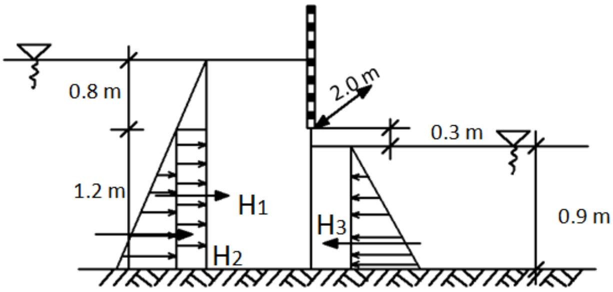
b. Find the direction and the magnitude of the force that should be applied to point B to hold the gate in position if there is oil inside the parts.

Answer: F_{water}=12.85 kN, F_{oil}=10.28 kN

$(\gamma_{oil} = 7.85 kN/m^3)$



Solution 3:



$H_{1water} = 9.81x0.8x1.2x2 = 18.83kN$

$H_{1oil} = 7.85x0.8x1.2x2 = 15.07kN$

$l_1 = \frac{1}{2}X1.2 = 0.6$

$H_{2water} = \frac{1}{2}\text{x}9.81\text{x}1.2\text{x}1.2\text{x}2 = 14.13\text{kN}$

$H_{2oil} = \frac{1}{2}x7.85x1.2x1.2x2 = 11.30kN$

$l_2 = \frac{2}{3}x1.2 = 0.8$

$H_{3water} = \frac{1}{2}\text{x}9.81\text{x}0.9\text{x}0.9\text{x}2 = 7.95\text{kN}$

$$H_{3oil} = \frac{1}{2}x7.85x0.9x0.9x2 = 6.38kN$$

$$l_3 = 0.3 + \frac{2}{3}x0.9 = 0.9$$

$$\sum M_A = 0;\ \ for\ water$$

$$H_{1water}xl_1 + H_{2water}xl_2 - H_{3water}xl_3 - 1.2xF = 0$$

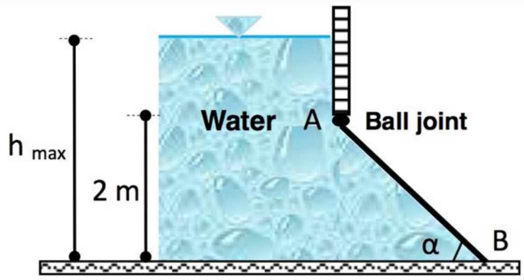
$$F_{water} = 12.85kN$$

$$\sum M_A = 0;\ \ for\ oil$$

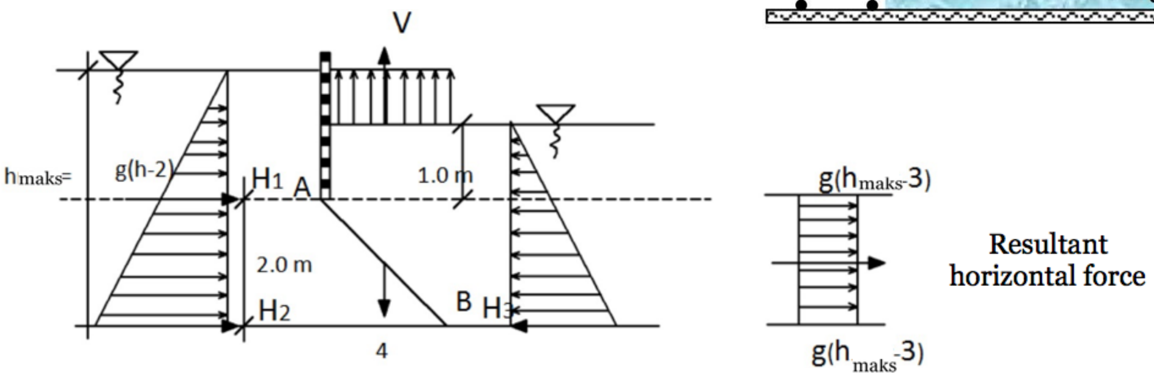
$$H_{1oil}xl_1 + H_{2oil}xl_2 - H_{3oil}xl_3 - 1.2xF_{oil} = 0$$

$$F_{oil} = 10.28kN$$

Question 4: The width of the rectangular gate AB in the figure that is hinged at point A is 4m, and it's weight is 392.4 kN. What should be the depth (h) of the water on the right side of the chamber to hold the gate in position? **Answer: h_{max}=5.5 m**



Solution 4:



$$H_1 - H_3 = \gamma x(h_{maks} - 2) - \gamma x1 = \gamma x(h_{maks} - 3)$$

$$H_2 - H_4 = \gamma xh_{maks} - 3x\gamma = \gamma x(h_{maks} - 3)$$

$$\sum M_A = -[\gamma x(h_{maks} - 3)x2x4x1] - [\gamma x(h_{maks} - 3)x2x4x1] + 392.4x1 = 0$$

$$2x(h_{maks} - 3) = 5$$

$$h_{maks} = 5.5m$$

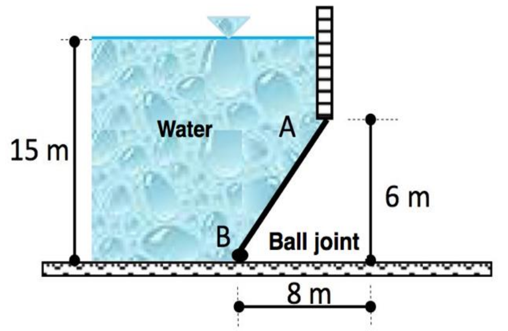
Pressure Forces

Question 5: For the square gate system in the drawing find:

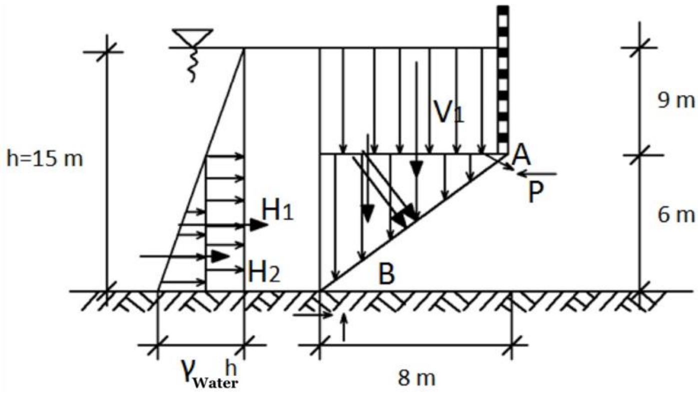
a- Pressure force acting upon the gate and the application point.

b- Reaction force at points A and B.

(The width of the gate perpendicular to the figure plane is 5 m. The contact force at point A is polished). **Answer:** **a) F_{water}=5886 kN, application point B , 4.58 m**
b- H_A=4495.92 kN H_B=-212.88 kN, V_B=3531.60 kN



Solution 5:



$$V_1 = \gamma_{water} x 9 x 8 x 5 = 3531.60 kN$$

$$V_2 = \frac{1}{2} x \gamma_{water} x 8 x 6 x 5 = 1177.20 kN$$

$$H_1 = \gamma_{water} x 6 x 9 x 5 = 2648.70 kN$$

$$H_2 = \frac{1}{2} x \gamma_{water} x 6 x 5 x (15-9) = 882.90 kN$$

Distance of each force to point B (horizontal, vertical):

$$l_{v1} = 4m, l_{v2} = \frac{8}{3} m, \qquad l_{H1} = 3m, l_{H2} = 2m$$

Resultant force ;

$$F = \sqrt{\left(\sum H_i\right)^2 + \left(\sum V_i\right)^2} \rightarrow F = \sqrt{(3531.60 + 1177.20)^2 + (2648.70 + 882.90)^2}$$

$$F = 5886 kN$$

$$Fxy_m = 3531.60x4 + 1177.20x\frac{8}{3} + 2648.70x3 + 882.90x2$$

$$y_m = \frac{26977.50}{5886} = 4.58m\ (Distance\ from\ the\ point\ B\ along\ the\ gate)$$

Reaction force P at point A,

$$\sum M_B = 0;$$

$$-Fxy_m + px6 = 0$$

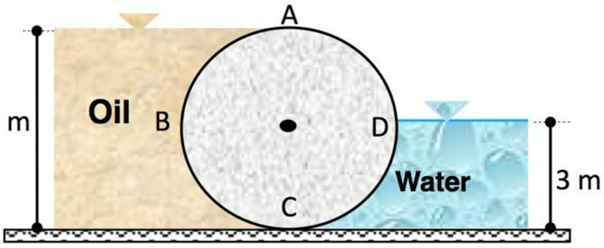
$$P = \frac{Fxy_m}{6} = \frac{5886x4.58}{6} = 4492.98kN$$

$$\sum X = 0 \rightarrow -P + H_B + Fsin\alpha = 0 \ \rightarrow -4492.98 + H_B + 5886X\frac{6}{10} = 0 \rightarrow H_B = 961.38kN$$

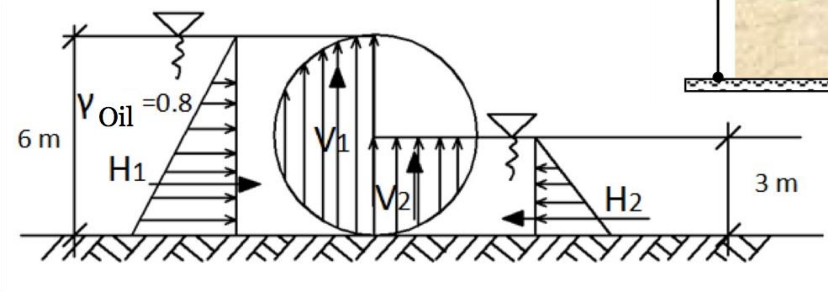
$$\sum Y = 0 \rightarrow \ V_B = Fcos\alpha \rightarrow V_B = 5886x\frac{8}{10} \rightarrow V_B = 4708.80kN$$

Question 6: Taking the width of the cylindrical gate perpendicular to figure plane as 1 m, find the horizontal and vertical components of the force and magnitude of the resultant force acting upon the gate and the coordinates of application point with respect to point A. ($\gamma_{oil} = 7.85kN/m^3$)

Answer: R= 204.83 kN (Resultant force)



Solution 6:



$$H_1 = \frac{1}{2}x\gamma_{oil}x6^2x1 = 141.26kN$$

$$V_1 = \frac{1}{2}x\frac{\pi x6^2}{4}x7.85x1 = 110.98kN$$

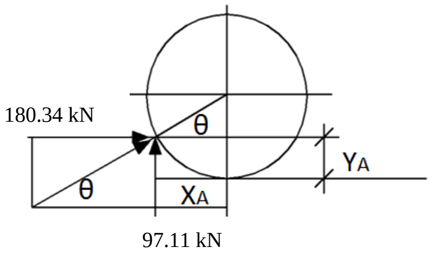
$$H_2 = \frac{1}{2}x9.81x3^2x1 = 44.15kN$$

$$V_2 = \frac{1}{4} x \frac{\pi x 6^2}{4} x 9.81 x 1 = 69.36 kN$$

$$\sum H = 141.26 - 44.15 = 97.11 kN$$

$$\sum V = 110.98 + 69.36 = 180.34 kN$$

Since the gate is cylindrical resultant force acts through the center of the circular region.

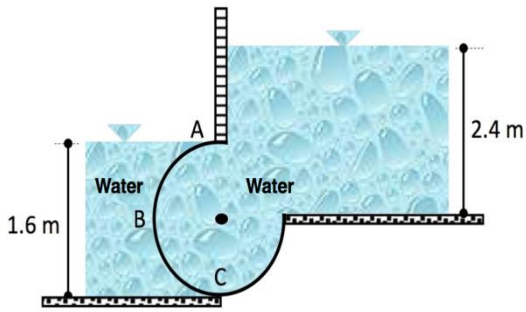


$$tan\theta = \frac{180.34}{97.11} = 1.85 \rightarrow \theta = 61.69°$$

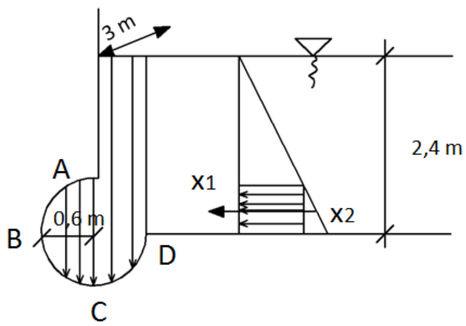
$$x_A = 3xcos\theta = 1.423m$$
$$y_A = 3 - 3xsin\theta = 0.36m$$

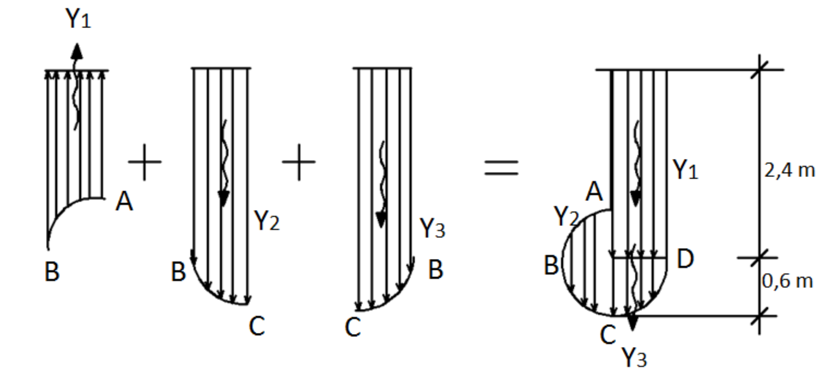
Question 7: Find the horizontal and vertical components of the force acting upon the curvilinear surface ABCD shown on the figure. (Width perpendicular to the figure plane is 3 m).

Answer: **$F_{horizontal}$=37.08 kN, $F_{vertical}$=67.59 kN**



Solution 7:

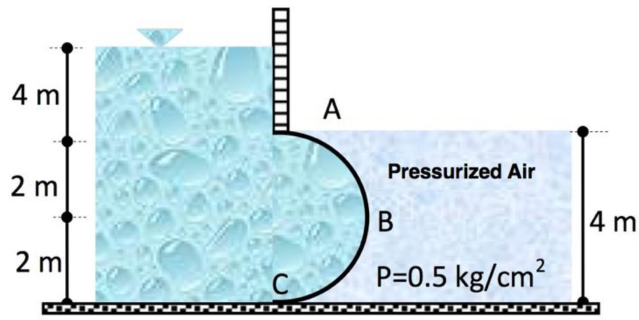




$Y_1 = 2.4x0.6x3x9.81 = 42.38kN$
$X_1 = 1.8x9.81x3 = 52.97kN$
$X_2 = 2.4x9.81x3 = 70.63kN$
$Y_2 = \frac{\pi x 0.6^2}{2} x3x9.81 = 16.68kN$

$Y_3 = \frac{\pi x 0.6^2}{4} x3x9.81 = 8.53kN$
$\sum X = 37.08kN$
$\sum Y = 67.59kN$
$F = \sqrt{x^2 + y^2} = 77.09kN$

Question 8: The width of the semi – cylindrical gate ABC (perpendicular to the figure plane) shown in the figure is 5 m. One side of the gate is pressurized air. Find the horizontal and vertical components of the force acting upon the gate. **Answer: $F_{horizontal}$=195.20 kN, $F_{vertical}$=308.23 kN**



Solution 8:

Vertical forces:

$V = \frac{\pi}{2} x2^2 x9.81x5 = 308.23kN$

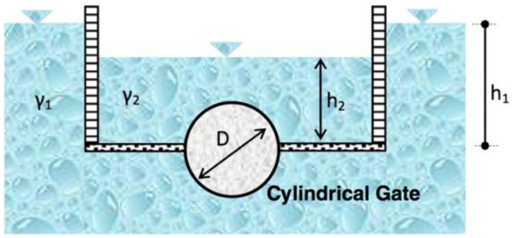
Horizontal forces:
$H = H_1 + H_2 - H_3$
$H_1 = 4x4x5x9.81 = 784.8kN$
$H_2 = 4x\frac{4}{2}x9.81x5 = 392.4kN$
$H_3 = 5x4x49.1 = 982kN$
$H = 784.8 + 392.4 - 982 = 195.2kN$

If a pressure force acts on a fluid at gas state in a closed container, the pressure is the same at every point.

Pressure Forces

Question 9: Find the pressure force acting on the cylindrical gate for the given chamber system. For the state of equilibrium, calculate the height h_2 in terms of the other parameters.

Answer: $h_2 = h_1\left(\frac{\gamma_1}{\gamma_2}\right)+\left(\frac{\pi.d}{8}\right)x\left(1+\frac{\gamma_1}{\gamma_2}\right)$



Solution 9:

No net horizontal force acts because the forces in the left and right directions cancel out.

$$V_1 = \left(h_2 d - \frac{\pi d^2}{8}\right)\gamma_2$$

$$V_2 = \left(h_1 d - \frac{\pi d^2}{8}\right)\gamma_1$$

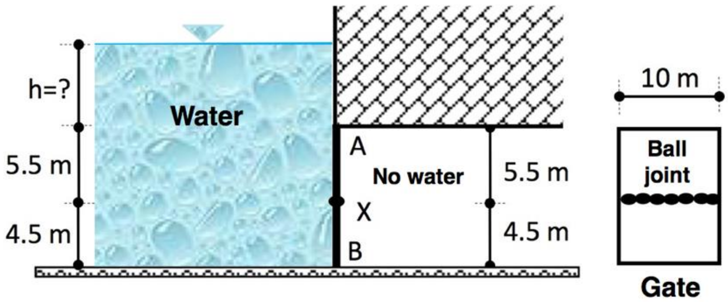
$$\sum F = V_1 + V_2$$

For equilibrium, it should be: $V_1 = V_2$. $h_2 d\gamma_2 - \frac{\pi d^2}{8}\gamma_2 = h_1 d\gamma_1 - \frac{\pi d^2}{8}\gamma_1$

$$h_2 = h_1\frac{\gamma_1}{\gamma_2} + \frac{\pi d}{8}\left(1+\frac{\gamma_1}{\gamma_2}\right)$$

Question 10: What should be the depth of the water so that the square designed butterfly damper could be opened. **Answer: h≤11.66 m (condition to stay closed)**

Solution 10:



$$H_1 = hx10x10x\gamma_{su} = 981h$$

$$H_2 = \frac{10x10}{2}x10x\gamma_{water} = 4905kN$$

$$M_x = 0$$

$$-H_1x0.5 + H_2x\left(4.5-\frac{10}{3}\right) = 0$$

$$-490.50h + 5722.50 = 0$$

$$h = 11.67m$$

$$h > 11.6m$$

Note: For the critical equilibrium condition, acting point of the resultant force has to be on the gate's pin. The same result could have been achieved by taking the moment with respect to point B.