Fluid Mechanics

Abdusselam Altunkaynak

1. Unit systems

1.1 Introduction

Natural events are independent on units.

The unit to be used in a certain variable is related to the advantage that we get from it.

In Newtonian Mechanics,

$$\vec{F} = m.\vec{a} = m.\frac{d\vec{v}}{dt} = m.\frac{dL}{dt^2}$$

In terms of units, the above equation can be rearranged as follows and the result should yield one.

$$\frac{[M].[L]}{[F].[T]^2} = M^0.L^0.T^0 = 1$$

There are different types of unit systems

A. The SI unit system

In this system,

Distance is given in meter (M), mass is given in Kilogram (Kg) and time is given in Second (s).

Therefore, in this system, the unit of force is as:

$$[F] = \frac{Kg.m}{s^2} = N$$

This unit is termed as Newton (N).

TABLE 1–1

The seven fundamental (or primary) dimensions and their units in SI

Dimension	Unit
Length	meter (m)
Mass	kilogram (kg)
Time	second (s)
Temperature	kelvin (K)
Electric current	ampere (A)
Amount of light	candela (cd)
Amount of matter	mole (mol)

TABLE 1–2

Standard prefixes in SI units

Multiple	Prefix
10^{24}	yotta, Y
10^{21}	zetta, Z
10^{18}	exa, E
10^{15}	peta, P
10^{12}	tera, T
10^{9}	giga, G
10^{6}	mega, M
10^{3}	kilo, k
10^{2}	hecto, h
10^{1}	deka, da
10^{-1}	deci, d
10^{-2}	centi, c
10^{-3}	milli, m
10^{-6}	micro, μ
10^{-9}	nano, n
10^{-12}	pico, p
10^{-15}	femto, f
10^{-18}	atto, a
10^{-21}	zepto, z
10^{-24}	yocto, y

B. The CGS system

In this system,

Distance is given in centimeter (cm) mass is given in grams (g) and time is given in second.

Therefore, the unit of force in this system is given as:

$$[F] = \frac{g.cm}{s^2} = Din.$$

This unit is termed as Din.

C. The MKfS system

In this system,

Distance is given in meter and time is given in seconds.

However, force is given in the form of kilogram force (Kgf). In this regard, the unit if mass will have the following form.

$$[M] = \frac{Kgf.s^2}{m}$$

D. The British (English) system

Here,

Distance is given in Foot (F), Force is given in Pound (Lb) and time is given in Minute (min).

Therefore,

$$[M] = \frac{Pound.s^2}{ft}$$

The English system is not considered as a good system.
This is because Force (Weight) is selected as basic dimension.
However,
weight is not a single variable that explains a bod
y as it is related to gravitational acceleration.

Remarks:

- *If we do not know the unit of a variable, we can develop its unit from the units of variables involved in defining the variable.*
- *The unit of any dimension is determined after selecting the unit system and with the help of the definition of the variable.*

1.2 Dimensional Homogeneity

Equation can be based on the physical basis that they are called **Rational Mechanical Equations.**

In Engineering, there also popular equations called **Empirical Equations.**

There are assumptions in both of the above equations.

If the coefficient(s) in the equation are non-dimensional, the equations are called **Homogeneous.**

Assumptions and approximations have two different characteristics:

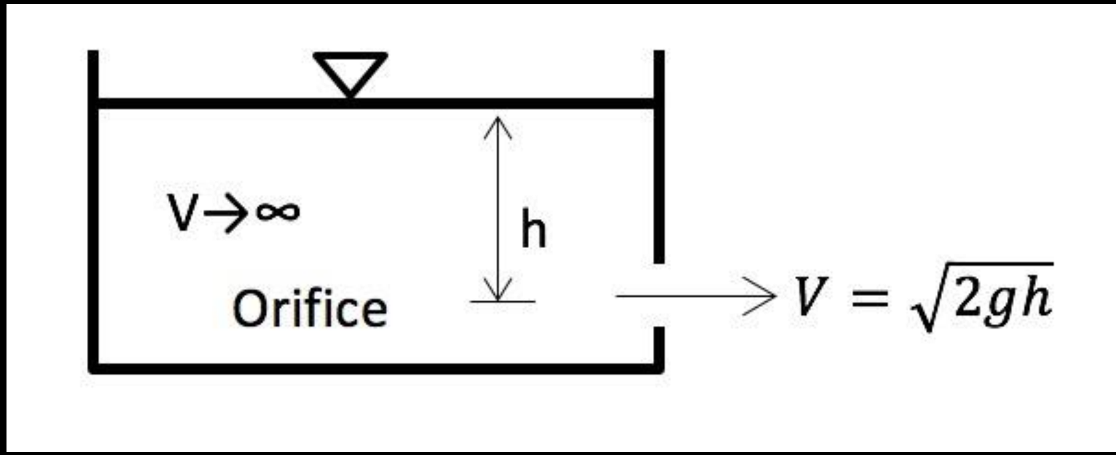
- *Assumptions made related to the physical properties of an environment. For example, we can assume fluid to be incompressible, frictionless fluids etc…..*

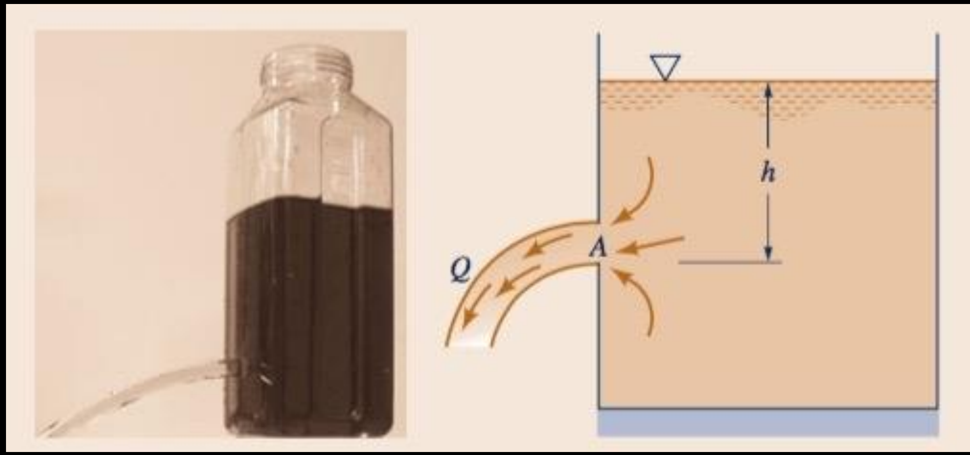
These kind of assumptions are related to physical characteristics

- *Approximations made in solving mathematical problems.*

For example, the use of series expansion in functions which are not integral.

For example, *the* ***velocity*** *of flow from orifices is given by* (Considering Bernoulli's equation and ideal flow condition):





Orifice

However, the actual velocity of flow in the flow is smaller than the ideal case because of energy loss as a result of the effect of frictional forces.

$V = C\sqrt{2g.h}$. As a result, C<1.

However, $[V] = [\sqrt{g.h}]$

This implies that

$[C] = M^0.L^0.T^0 = 1$

This shows that the coefficient is unit less and, therefore, the relationship is homogeneous.

- *In homogeneous equations, the coefficients are independent from unit systems.*
- *In hydraulic and water structures, equations generally have empirical nature and, therefore, they are not homogeneous.*
- *Because of this, the value of the coefficient to be used in a certain unit system should be explained initially.*

1.3 Concepts of Specific mass (specific gravity), Specific weight and Density.

Specific mass is defined as the ***mass (M) per unit volume*** of a body and it has a unit.

$$\rho = \frac{M}{\forall}$$

The specific weight is defined as the ***weight of a body per unit volume*** and it has a unit.

$$\gamma = \frac{W}{\forall}$$

The units of the specific mass and specific weight are related to the unit system used.

The density of a body (d) could be dimensional or non-dimensional depending on the definition used to describe it.

It is known that **weight** is the ***product of mass and gravity.*** Therefore,

$$\gamma = \rho . g$$

1.4 Solid mechanics, Fluid Mechanics and hydraulics

Solids and fluids have basic differences.
These differences are given in the following table.

Matter	*Distance between molecules*	*The molecules' ability to move freely*
Fluid	*Much more bigger*	*Much more bigger*
Solid	*Very small*	*Very small*

In analyzing fluids, the concept of *continuum mechanics* is used.

In this concept,

The region where the value of ***the specific mass is independent from the number of molecules*** is termed as a **continuum**.

If we draw and see the relationship between the specific mass and the number of molecules, we will find out that the specific mass tends to remain constant after a certain number of molecules.

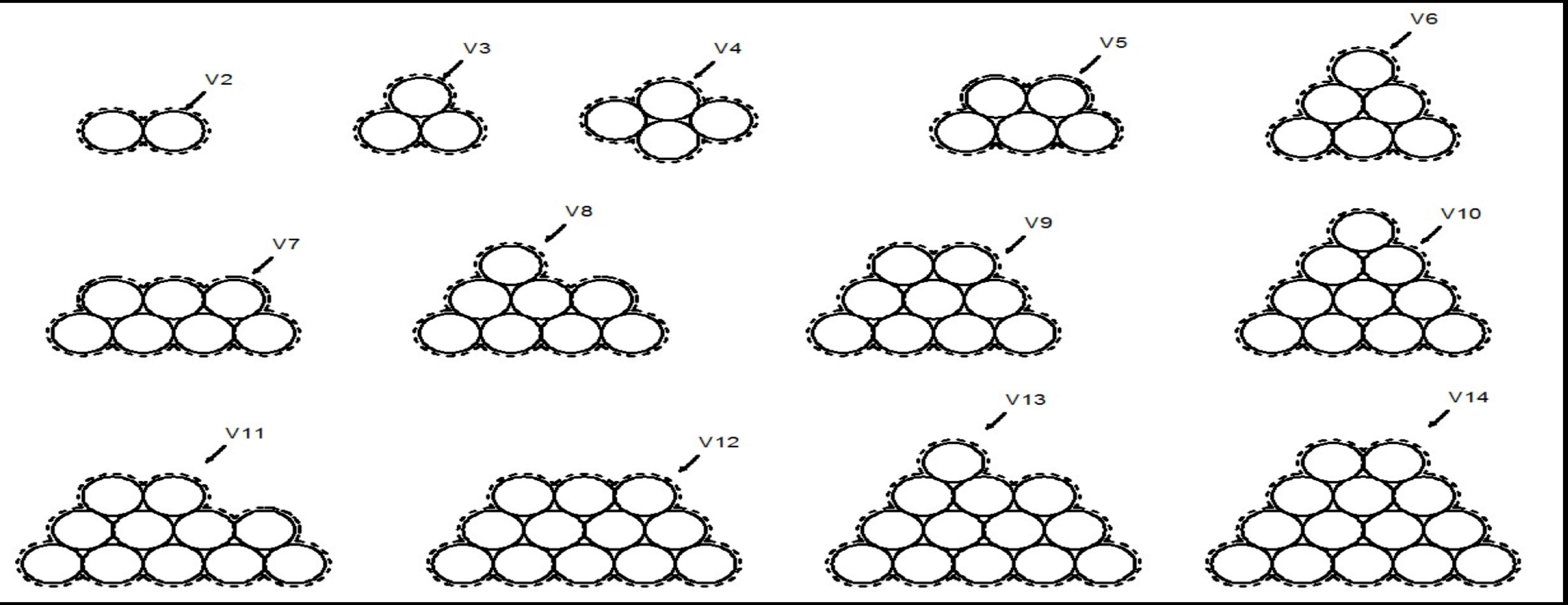


Fig.7: The relationship of specific mass and number of molecules

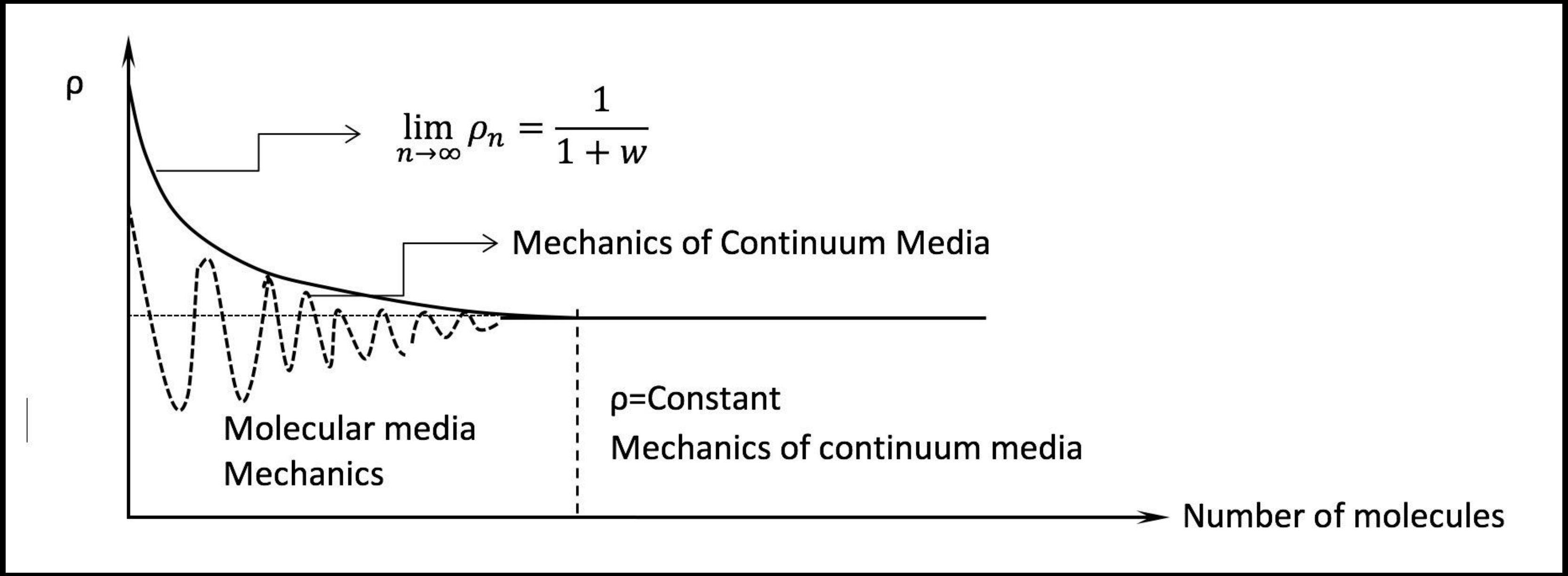


Fig.8: The relationship of specific mass and number of molecules

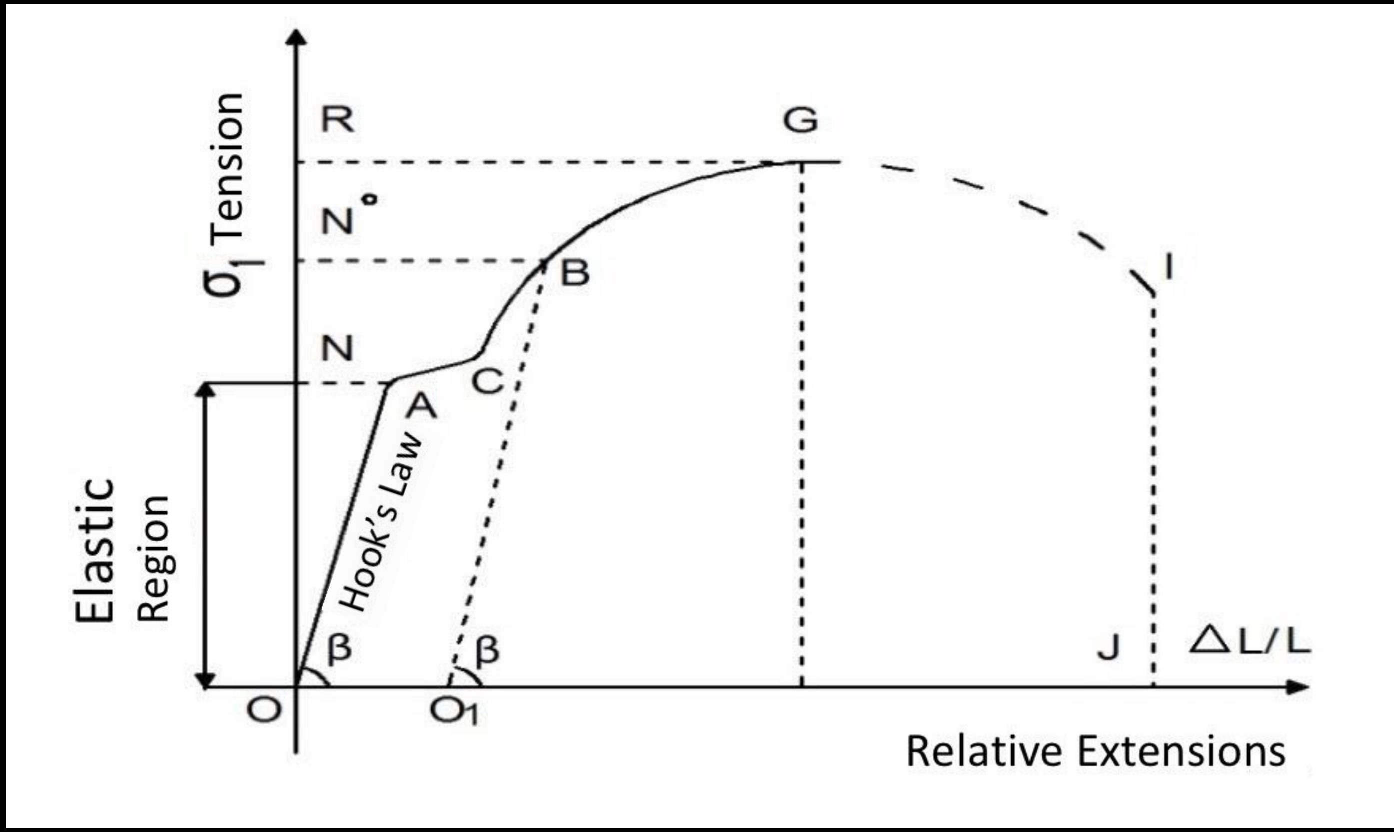
Definition:

Fluid is contentious, flowing material that, under a very small force, is deformed, changes its shape and, therefore, does not have its own shape.

In fluids, ***a decrease in the force*** → the ***slowdown in rate of deformation*** or change of shape.

Actually, in solids, the force required to result is change the shape is very huge.

This relationship is shown by Hook's law that show force and deformation to be directly proportional.



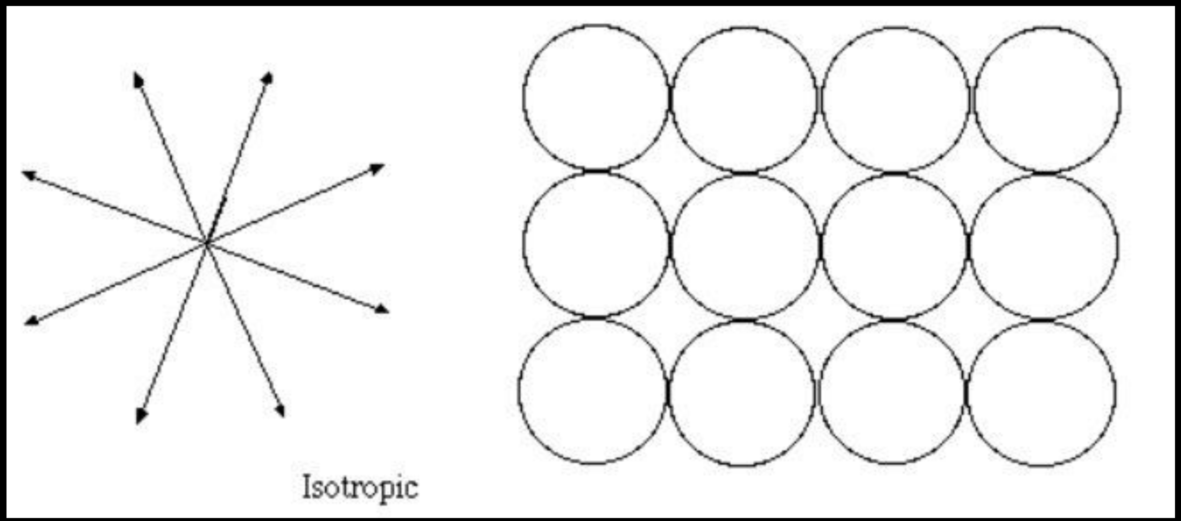
The Hook's law

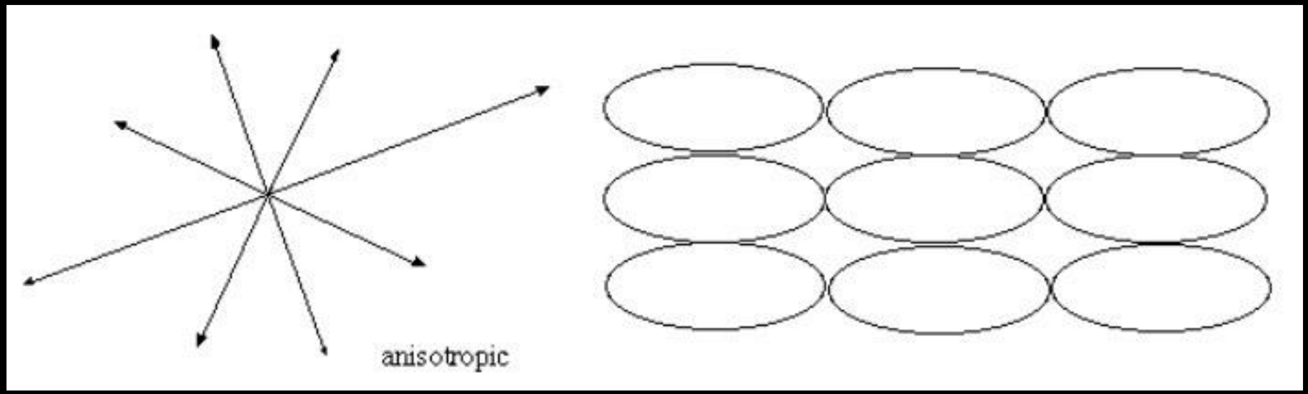
The most important properties of fluids.

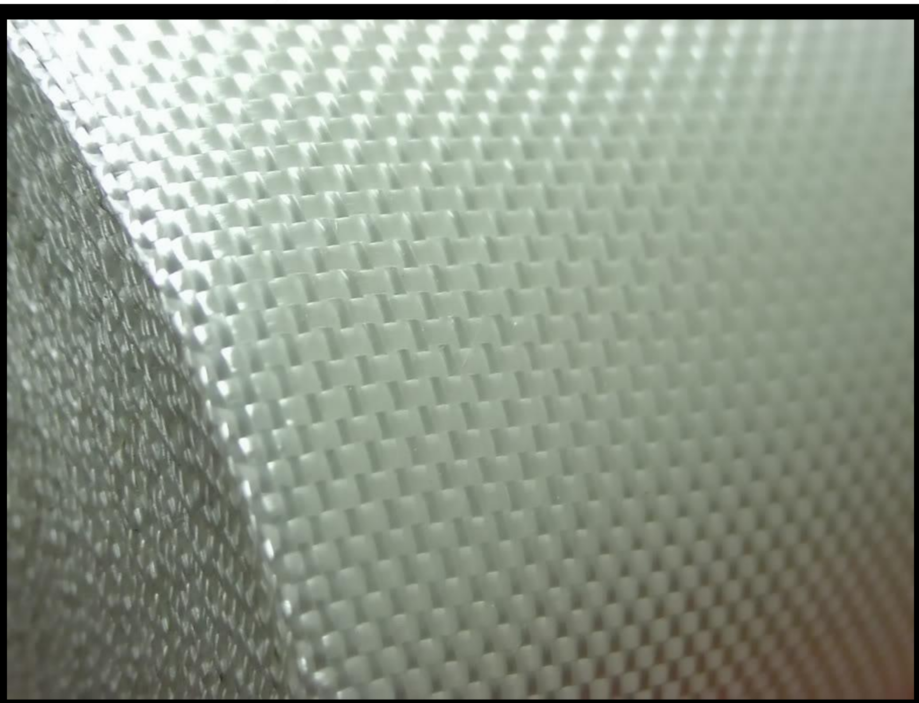
A. ***Isotropic***: *the values of physical parameters are not related to direction*

isotropic

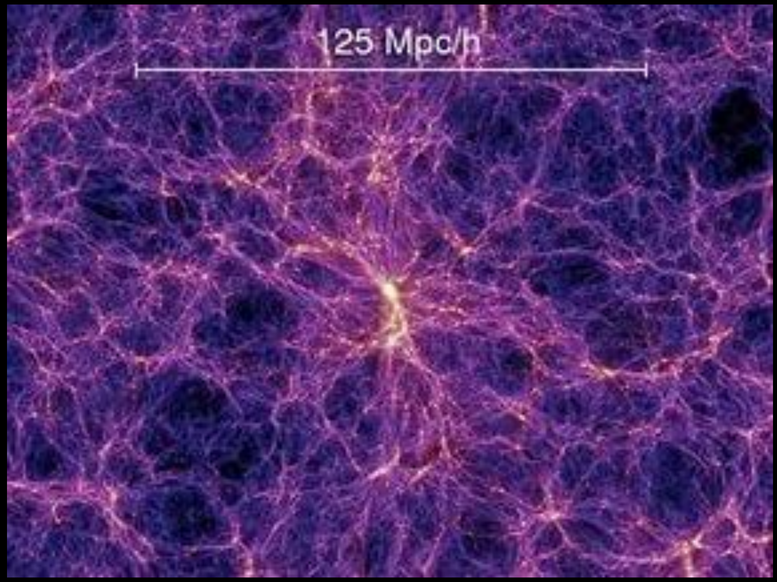
anisotropic







Isotropic Glass Fiber



Anisotropic space

B. **Viscosity**: *the resistance of the fluid against deformation or change in shape*

The behavior of a flowing fluid depends on various fluid properties. Viscosity, one of the important properties, is responsible for the shear force produced in a moving fluid. Although the two fluids shown look alike (both are clear liquids and have a specific gravity of 1), they behave very differently when set into motion. ***The very viscous silicone oil is approximately 10,000 times more viscous than the water***.

Dropping a 100g weight from 80 cm into 8 different beakers of liquid. **Testing the 'inner resistance' of each of the liquids or their 'thickness**

C. **Mobility**: Fluids do not have their own shape. They take the shape of the containers.
This effect of property shows acceleration
and it is the cause for the existence of the **non-linear term of acceleration.**

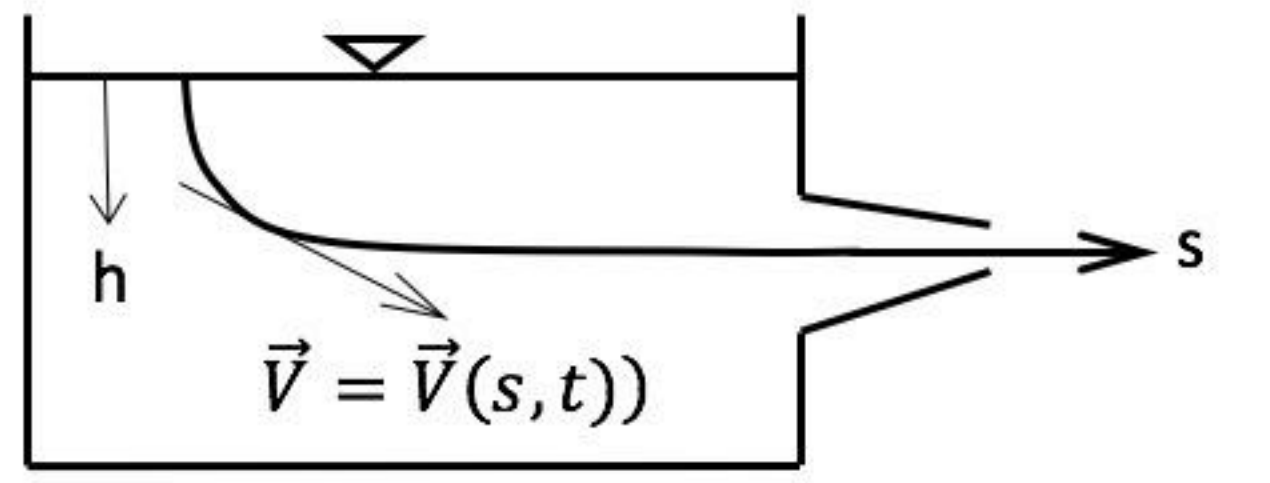


Fig.11: The orifice

This implies that:

$$d\vec{V} = \frac{\partial \vec{V}}{\partial t}.dt + \frac{\partial \vec{V}}{\partial s}.ds$$

If we divide both sides of the above equation by dt, we will find:

$$a = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \frac{ds}{dt}\frac{\partial \vec{V}}{\partial s} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\frac{\partial \vec{V}}{\partial s}$$

Notes:

- *The term* $\vec{V}\frac{\partial \vec{V}}{\partial s}$ *is called convective acceleration and it is the non-linear therm.*

  This term is not found in Solid mechanics.

- *Here, it should be noted that*

$$\frac{d\vec{V}}{dt} \neq \frac{\partial \vec{V}}{\partial t} \qquad \frac{d\vec{V}}{dt} = \frac{D\mathbf{V}}{Dt}$$

In mathematics:

A Definition of Linear term: In any term, if the coefficient of the dependent variable in the problem and/or the coefficient of the derivative is constant, the term is called **linear**.

If the coefficient is only related to the dependent variable of the problem, the term is called **queasy-linear**

If the coefficient is also related to the dependent variable of the problem, the term is called **non-linear**.

B Physical interpretation of exact differential:

If we have a certain cubic volume as an example:

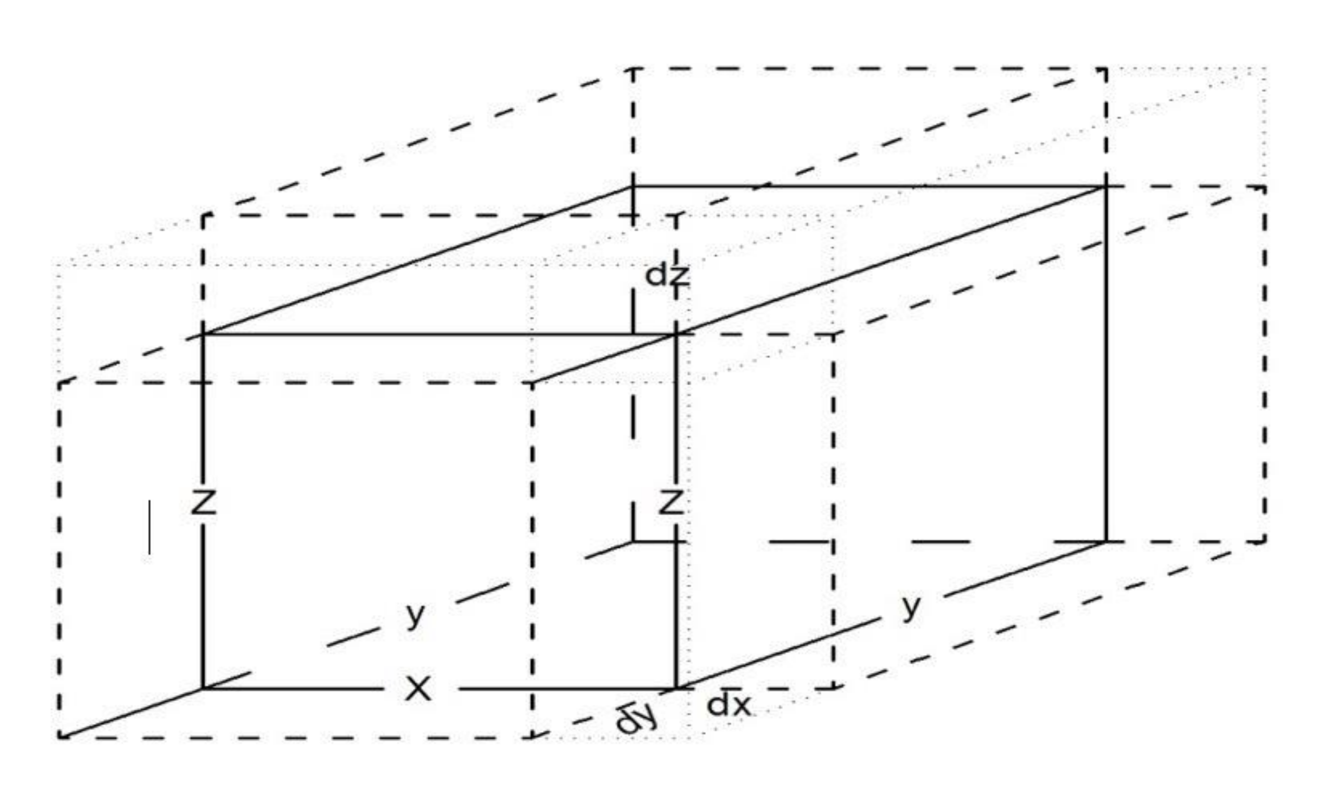


Fig.12: A cubic volume and its dimensions

the total change in Volume

$$d\forall = d\forall_x + d\forall_y + d\forall_z = \frac{\partial \forall}{\partial x}.dx + \frac{\partial \forall}{\partial y}.dy + \frac{\partial \forall}{\partial z}.dz$$

Remarks:

- *Fluid mechanics is **more complex** than solid mechanics*
- *It is less developed because:*
  - *The **non-linear convective acceleration** is the one that controls the problem in fluids*
  - *In terms of geometry, there are unknowns in the problems*

These have given birth to sciences related to

- *Fluid mechanics, which is developed based of the laws of rational mechanics and*
- *Hydraulics, which involves more errors as they are developed more on observational,*

synoptic and empirical approaches developed in order to solve current problems.

1.5 Observed Forces

If we consider a certain body having a certain volume and mass under a certain exerted force, considering Newtonian mechanics, we will have that:

$$\sum \vec{F} = m.\vec{a}$$

$$\vec{F} = \vec{N} + \vec{T}$$

$$\frac{\vec{F}}{A} = \frac{\vec{N}}{A} + \frac{\vec{T}}{A}$$

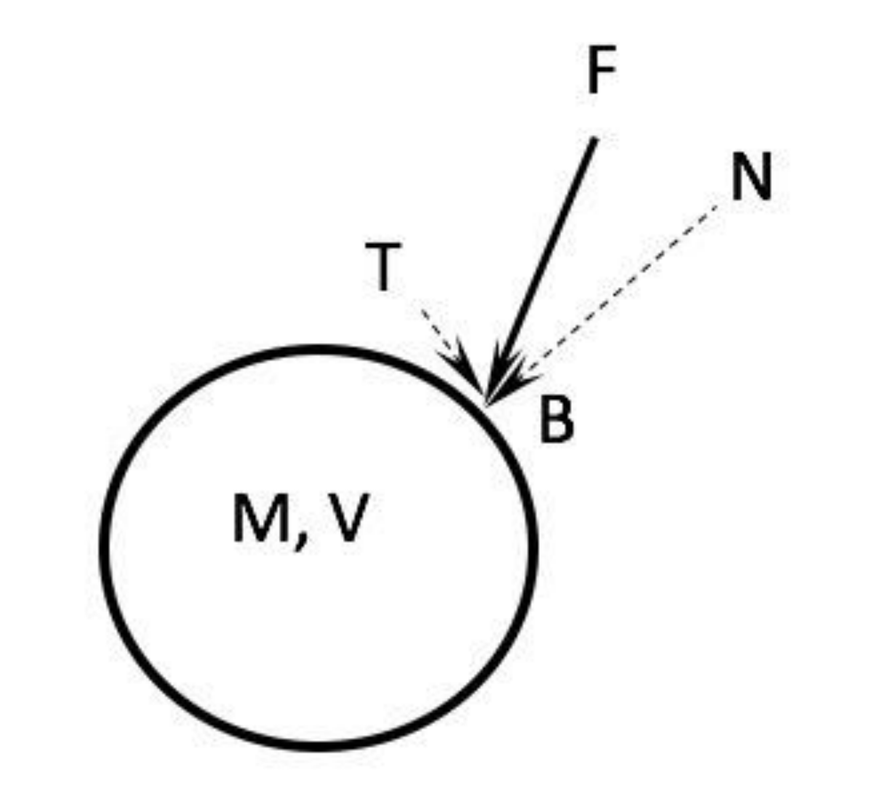


Fig.12: The external force acting on a spherical body

$\frac{\vec{N}}{A} = \sigma$ is called Normal stress and $\frac{\vec{T}}{A} = \tau$ is called Shear stress.

In addition,

The volumetric force which is the result of the weight of the body itself

is taken as another external force acting on the body.

$$F = m.g = \rho.\forall.g = \gamma\forall$$

$$m.\omega^2.r = \rho.\forall.\omega^2.r$$

These kinds of expressions represent volumetric or Body forces.

Remark:

There are types of external forces in analysis

- *Volumetric (body) forces as a result of the presence of a body and*
- *External forces exerted on the body: normal(perpendicular) to the surface and tangential to the Surface*

- ***According to the above explanations:***

$$\sum \vec{F} = m.\vec{a} = N\vec{F} + F\vec{F} + B\vec{F}$$

$B\vec{F}$ is the ***volumetric (body) force***, $N\vec{F}$ is ***pressure force*** acting normal to the surface and $F\vec{F}$ is ***the frictional (shear) force*** acting tangential to the surface.

1.6 Static, Kinematic and Dynamic Analysis

1. Static analysis

In static condition, there is no motion, i.e. velocity is zero. We know that the shear stress is a function of velocity given as:

$$\tau = \mu \frac{dv}{dy}$$

Since velocity is zero, the shear stress becomes zero.
Therefore, in static problems, only normal stress is available.

2. Kinematic Analysis

In this analysis, there is motion and therefore, $v \neq 0$. However, the forces that caused the motion and the forces created as a result of the motion (shear/frictional) forces are not considered.

3. Dynamic analysis

Here again, there is motion, but as opposed to Kinematic analysis,

the forces that caused the motion and forces created as a result of the motion are considered.

Therefore, all external forces (volumetric, normal and tangential forces) are considered.

The difficult thing in dealing with hydraulics is the determination of the tangential (shear) force.

Dynamic analysis is done under two categories:

1. *The dynamics of Ideal fluids*

In this fluids, there is no shear stress.

2. *The dynamics of real fluids*

In this condition, shear stress is not ignored.

1.7 Stresses at a point

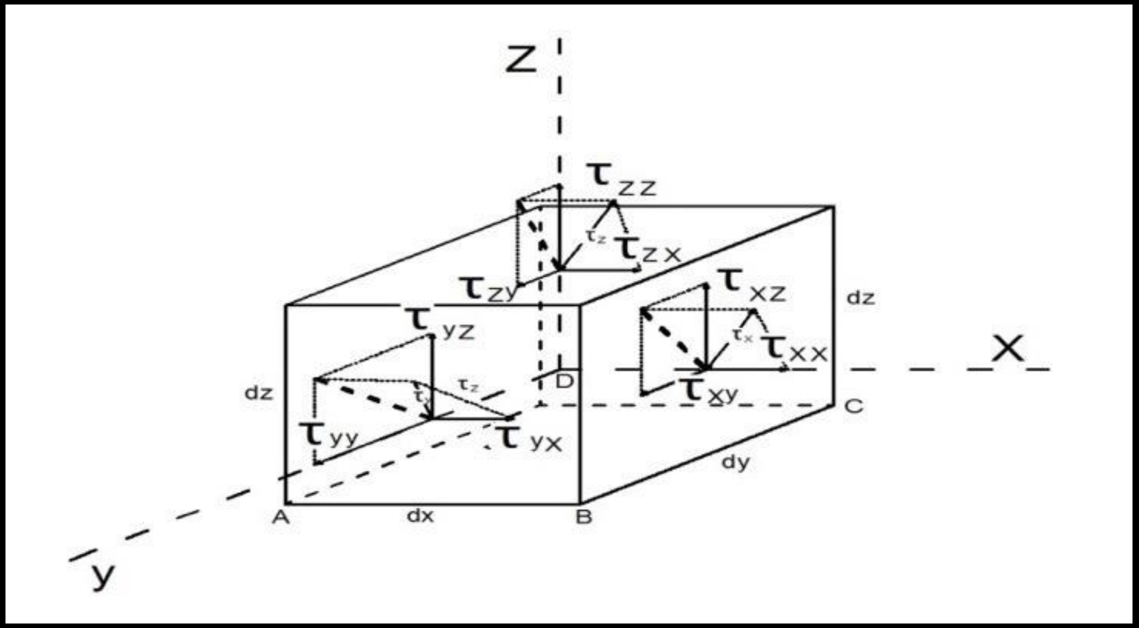


Fig.13: Stress acting at a point

If we consider a certain cubic control volume, normal and shear stresses act on all the surfaces of the control volume. In general, there are 9 components of stress acting at a point, and stress can be taken as a tensor written in the following form.

$$\begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}$$

Note:

- *A variable having only one value and not related to direction is termed as* ***Scalar***
- *A variable has three components at a point is called* ***vector*** *and*
- *A variable having nine components at a point is termed as* ***tensor***
- *From the above tensor, we can see that the stress at a point has 6 components having different values from one another because* $\tau_{yx} = \tau_{xy}, \tau_{zx} = \tau_{xz}$ and $\tau_{yz} = \tau_{zy}$ *and*
- *This means that we get values of 3 normal and 3 shear stress components at a point.*

1.8 Behaviors of fluids against stress

A. Behavior against shear stress:

Viscosity:

If we consider a flow in a certain channel, there is velocity variation as we move far from the wall of the channel. Obviously, the velocity decreases as we approach the wall. As a boundary condition, we can consider that at y=0 (at the wall), the velocity is zero (v=0).

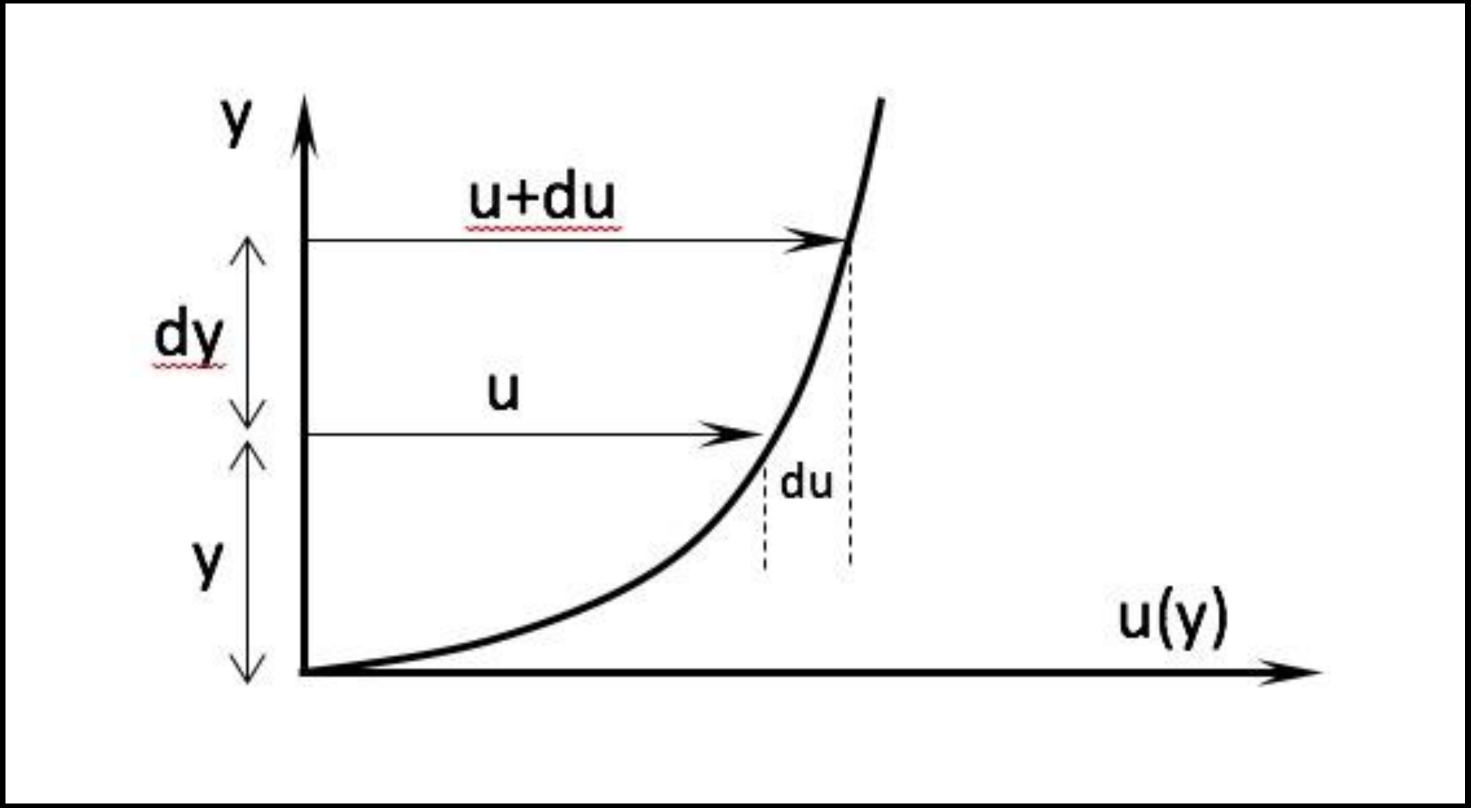


Fig.14: The velocity distribution along depth of flow

As a result of the difference in the velocity of layers of fluid, friction (shear stress) should be created. In simple terms,

$$\tau = \mu \frac{du}{dy}$$

This is Newton's elemental law of shear stress and it is given only for laminar flow conditions.

In the above equation:

- τ *refers to the shear stress created as a result of the friction between neighboring layers having different velocities*
- $\frac{du}{dy}$ *refers to velocity gradient.*
- *This means, if 'du' is the change in velocity for the change in location up to 'dy', the change in velocity for a unit change in location becomes* '$1\frac{du}{dy}$' .
- *As a result,* $\frac{du}{dy}$ *is a velocity gradient that shows the change in the 'u' component of the velocity of the fluid that occurs as a result of a unit change in location 'y' in the y direction.*
- *In the same manner,* $\frac{dP}{dy}$ *is a pressure gradient that shows the change in 'P' as a result of a unit change in location 'y' in the y direction.*

- μ *is a coefficient related to the type of fluids. It is a characteristic of the fluid that it shows against shear force. It is called Dynamic viscosity and it shows the property of the fluid against deformation.*
- *There is a viscosity called kinematic viscosity* (ϑ) *which is defined as* $\vartheta = \frac{\mu}{\rho}$ *.*
- *Both* μ *and* (ϑ) *are functions of pressure and temperature. However, in practice, the effect of pressure is ignored. It is known that fluid are divided into two classes based on the effect of temperature on the viscosity of the fluids. There are:*

- *Liquids: fluids where values of and decrease as temperature increases and*

- *Gases: fluids where values of and increase as temperature increases.*

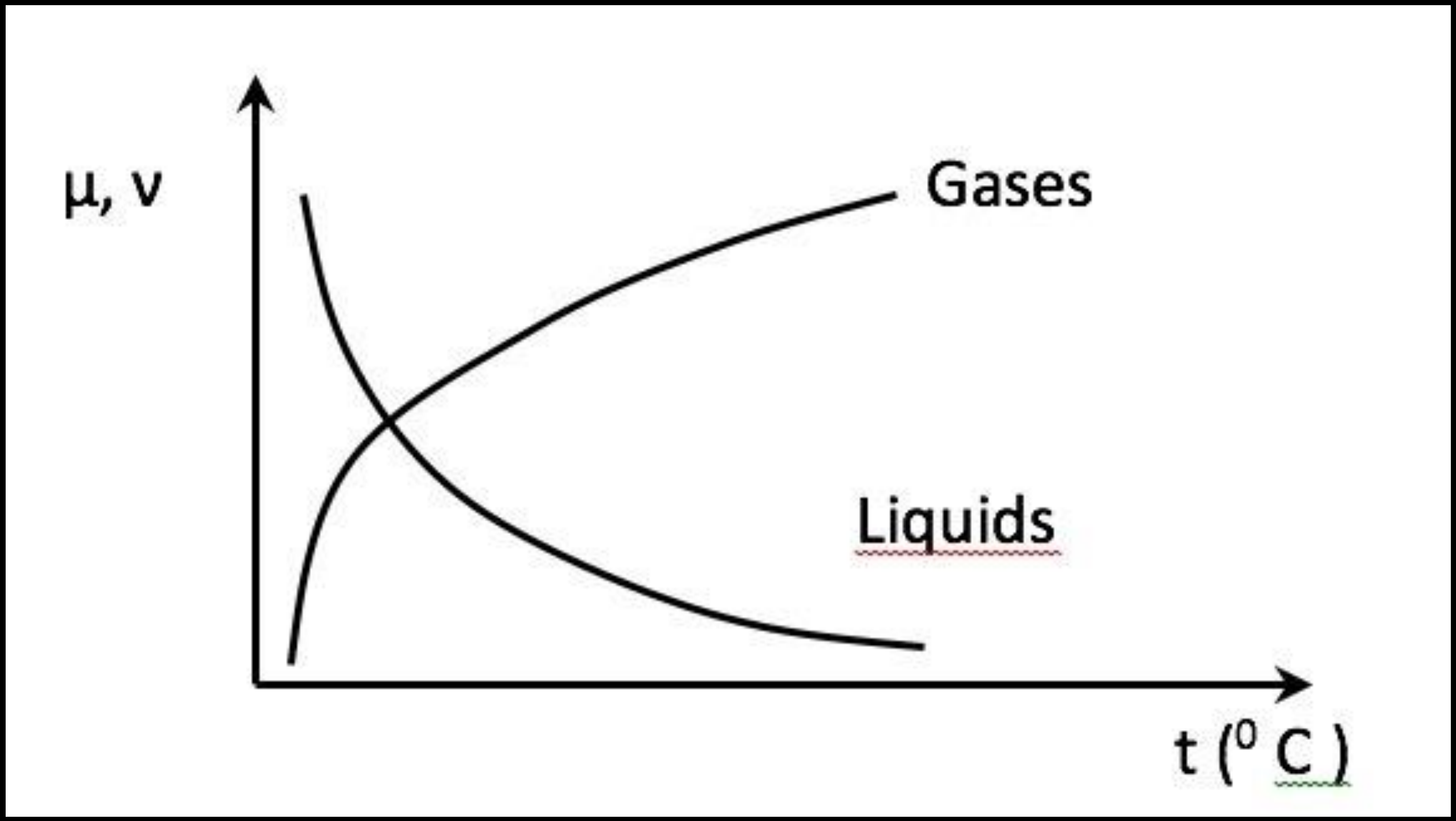


Fig.15: The relationship between viscosity and temperature

Note:

1. Fluids where the relationship $\tau = \mu \frac{du}{dy}$ is linear are called ***Newtonian fluids*** and fluids where $\tau = \mu \frac{du}{dy}$ is not non-linear, i.e. , are termed as ***non-Newtonian fluids***.

In this course, we will only deal with Newtonian fluids.

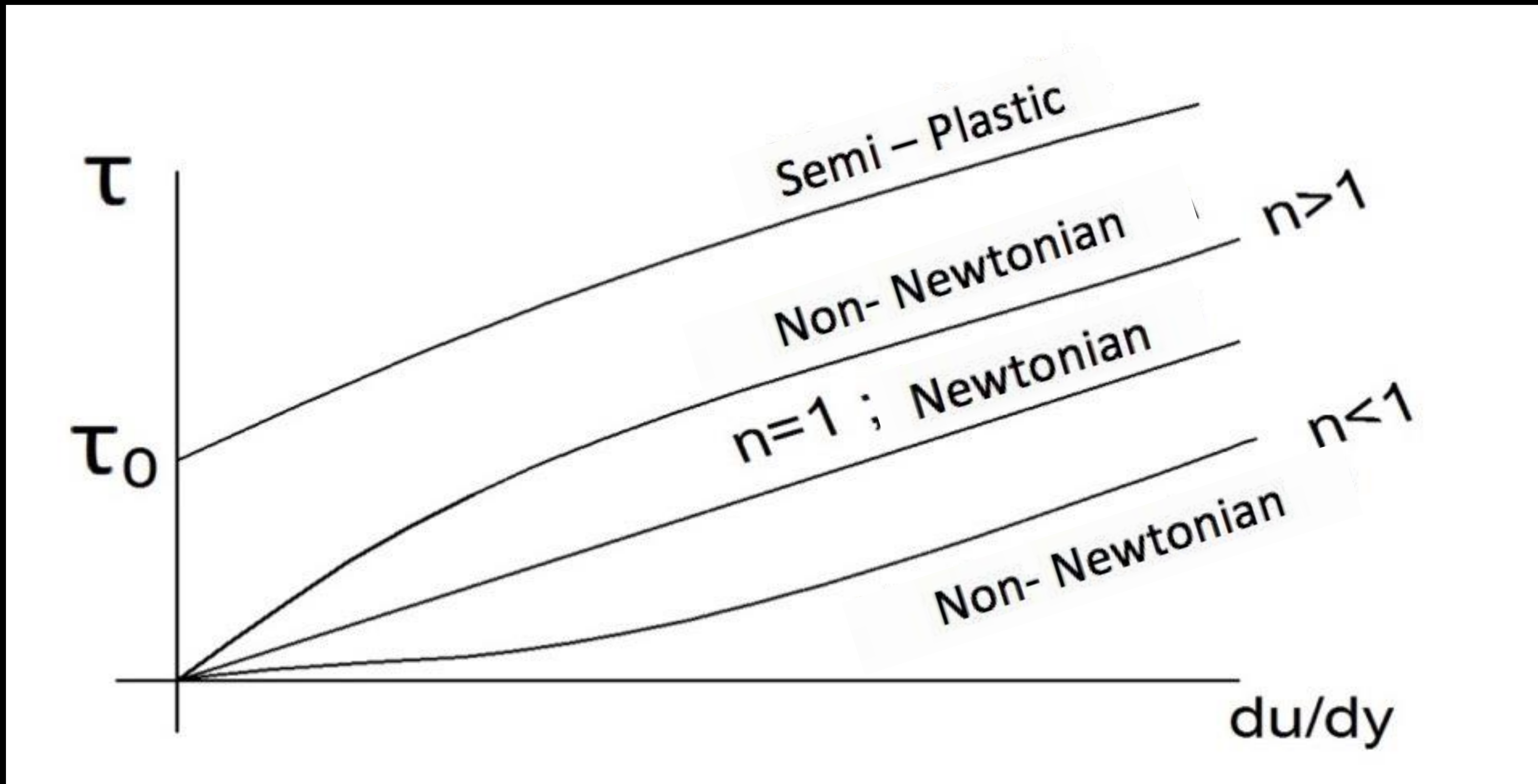
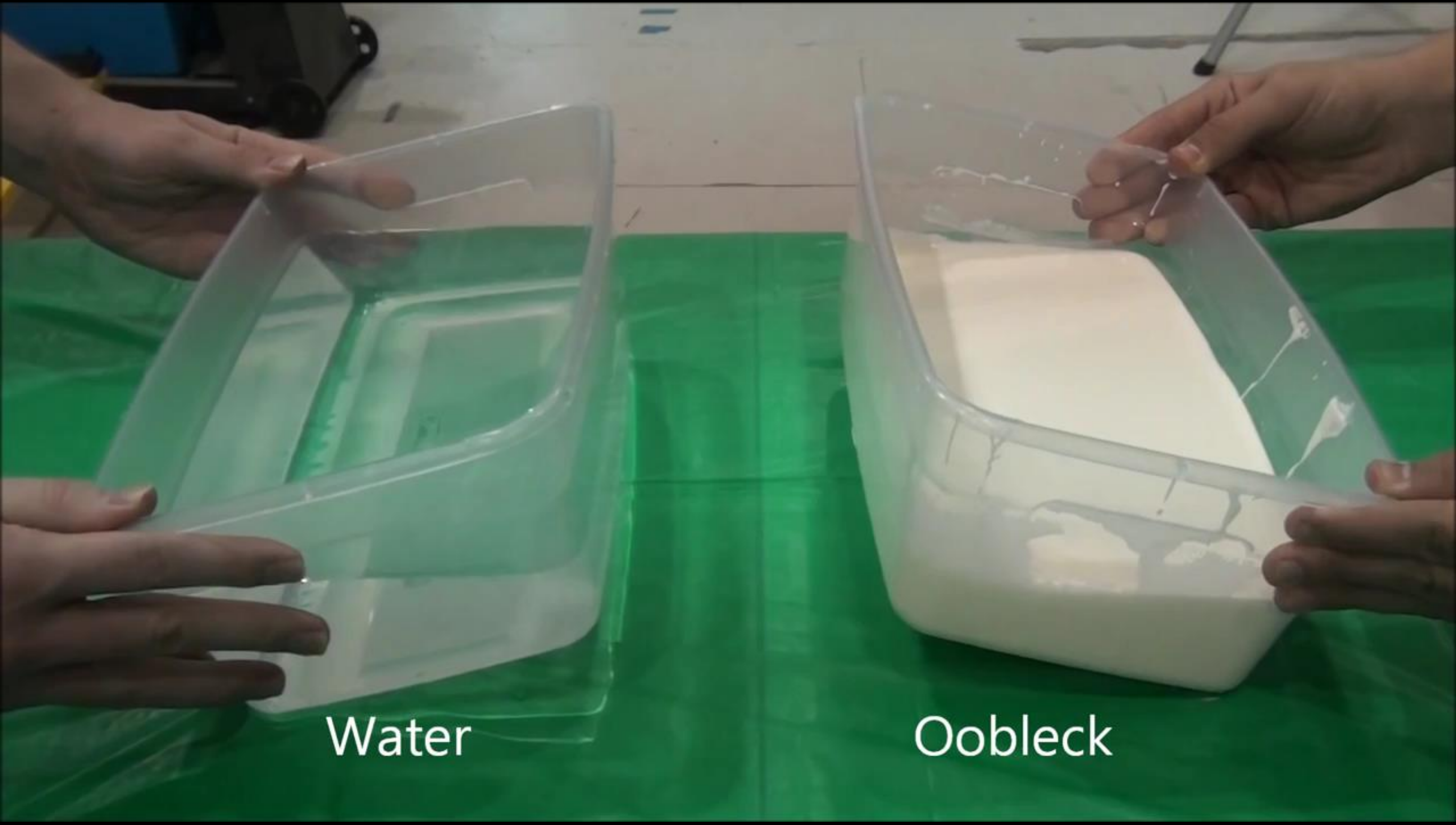


Fig.16: The relationship between τ and $\left(\frac{du}{dy}\right)^n$ *and for different values of n.*

In a Newtonian fluid, the force required for deformation is very small and the rate of deformation is directly proportional to force applied .

THERE IS NO τ IN IDEAL FLUIDS


Water
Oobleck

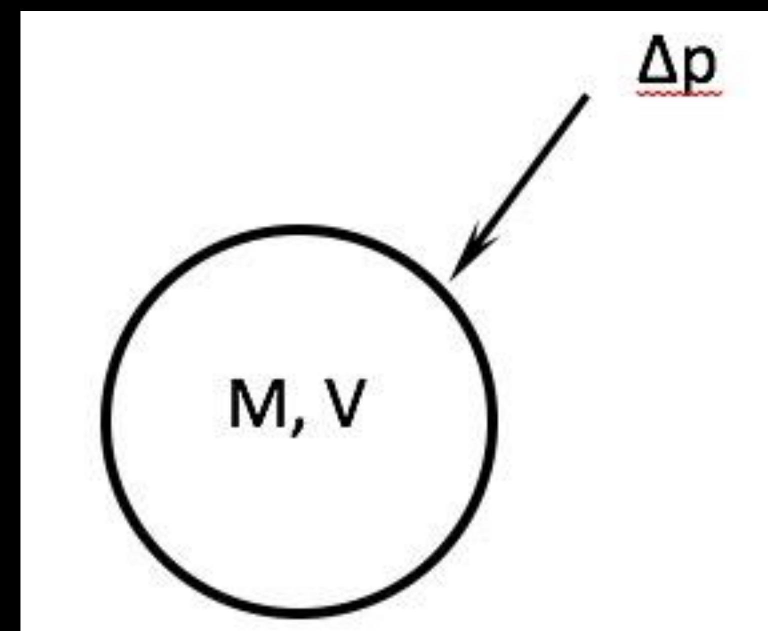
Corn starch is a shear thickening non-Newtonian fluid meaning
that it becomes more viscous when it is disturbed.
When it's hit repeatedly by something like a speaker cone it forms weird tendrils.

B. Behavior against normal stress:

Compressibility behavior:

Compressibility is a very important behavior. We know that elastic compressibility under pressure is given by:

$$\frac{\Delta V}{V} = -\frac{\Delta P}{\varepsilon}$$

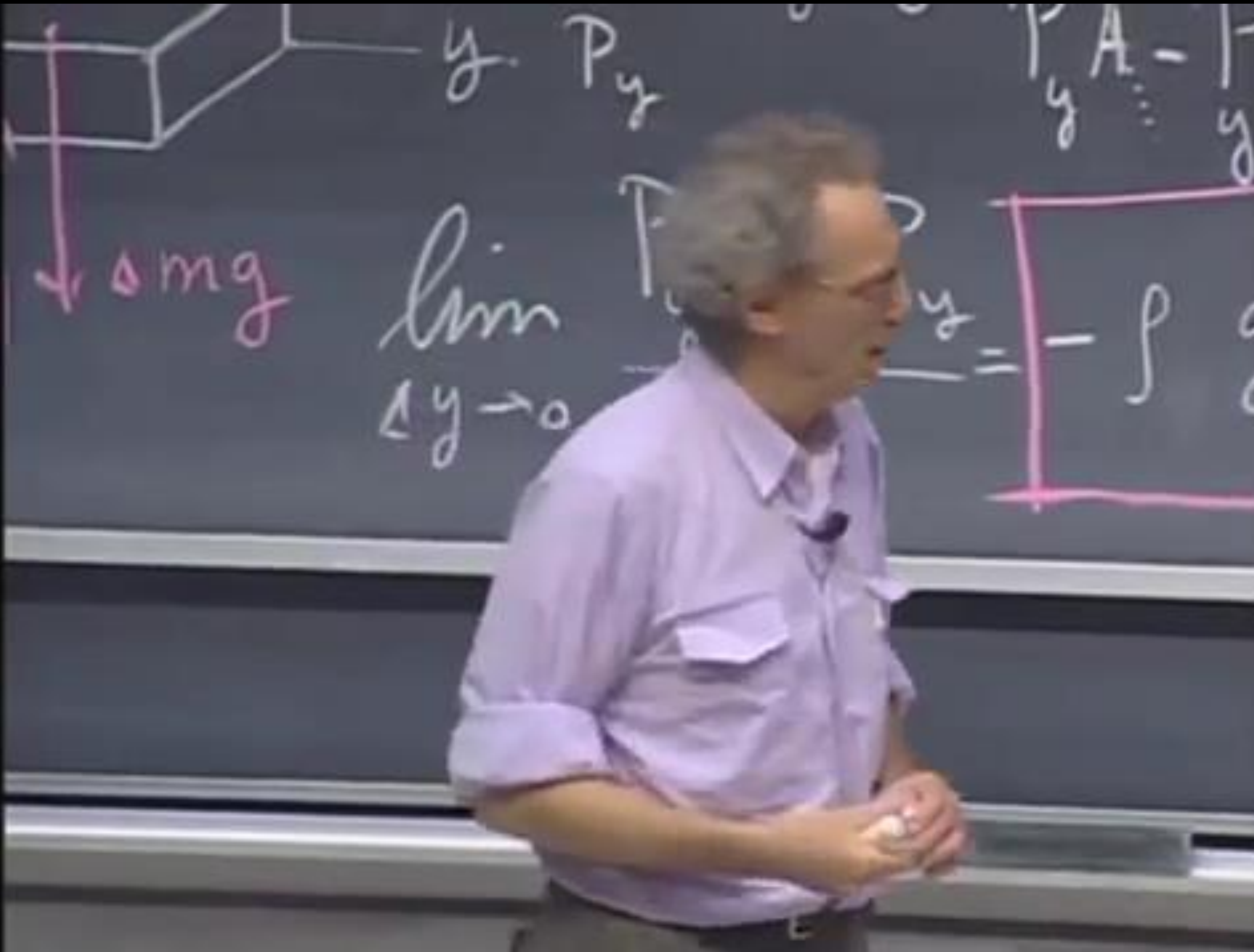


$$\varepsilon_{water} = 2x10^4\ Kgf/cm^2$$

$$\varepsilon_{iron} = 2x10^6\ Kgf/cm^2$$

ε_{gass} is very small

This shows that $\varepsilon_{iron} >> \varepsilon_{water} >> \varepsilon_{gass}$



y. P
mg
lim
Δy→0
A

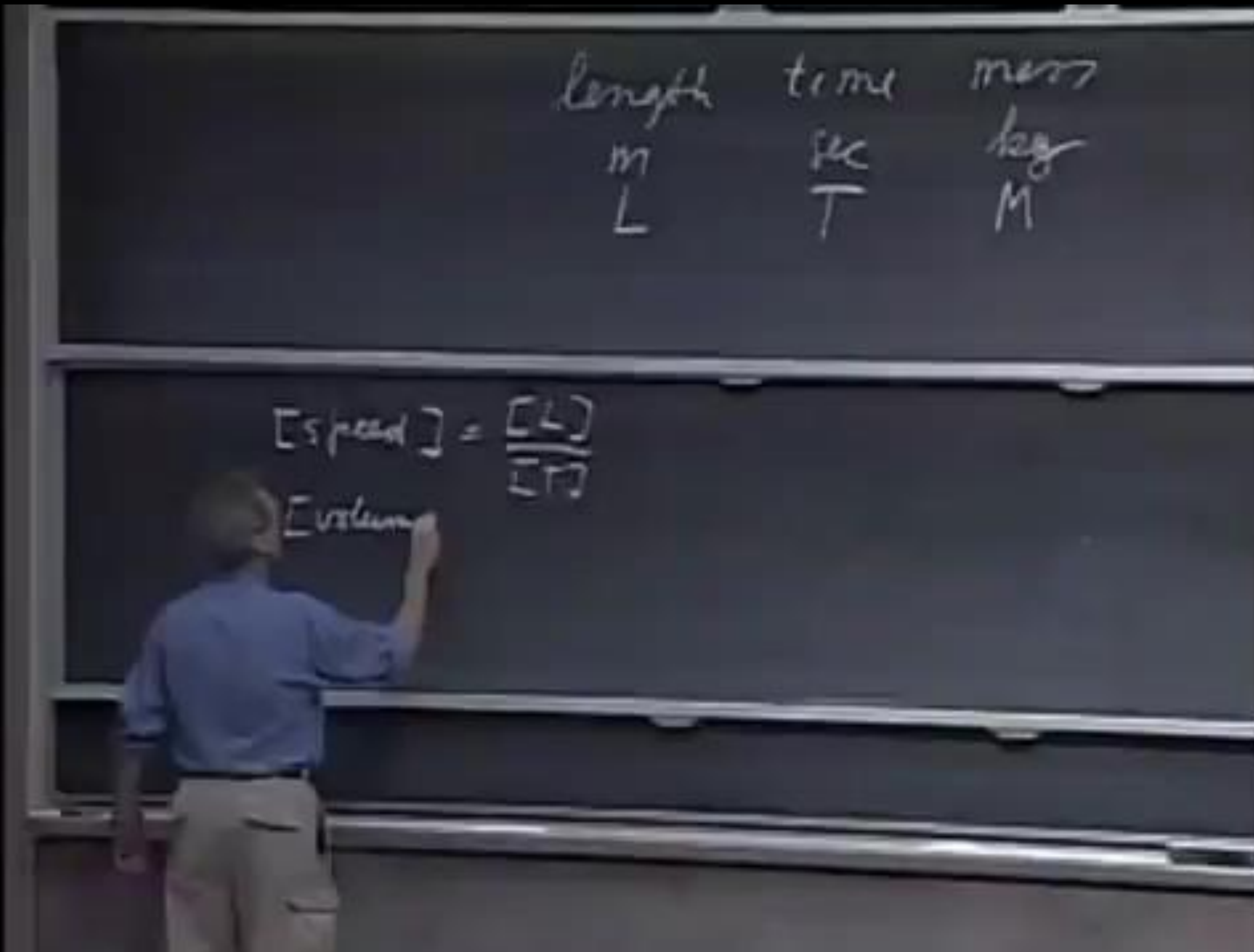
In general, liquids are taken as incompressible fluids.

In this course, Liquids will be considered as incompressible fluids.

*A pressure of 210 atm causes the density of liquid water at 1 atm to change by just **1 percent**.*

Taking compressibility into consideration and based on the behavior of fluids against normal stress, fluids can be divided into two classes:

Compressible fluids (gasses) and Incompressible fluids (Liquids).



length time mass
m sec kg
L T M
[speed] = [L] / [T]
[volum

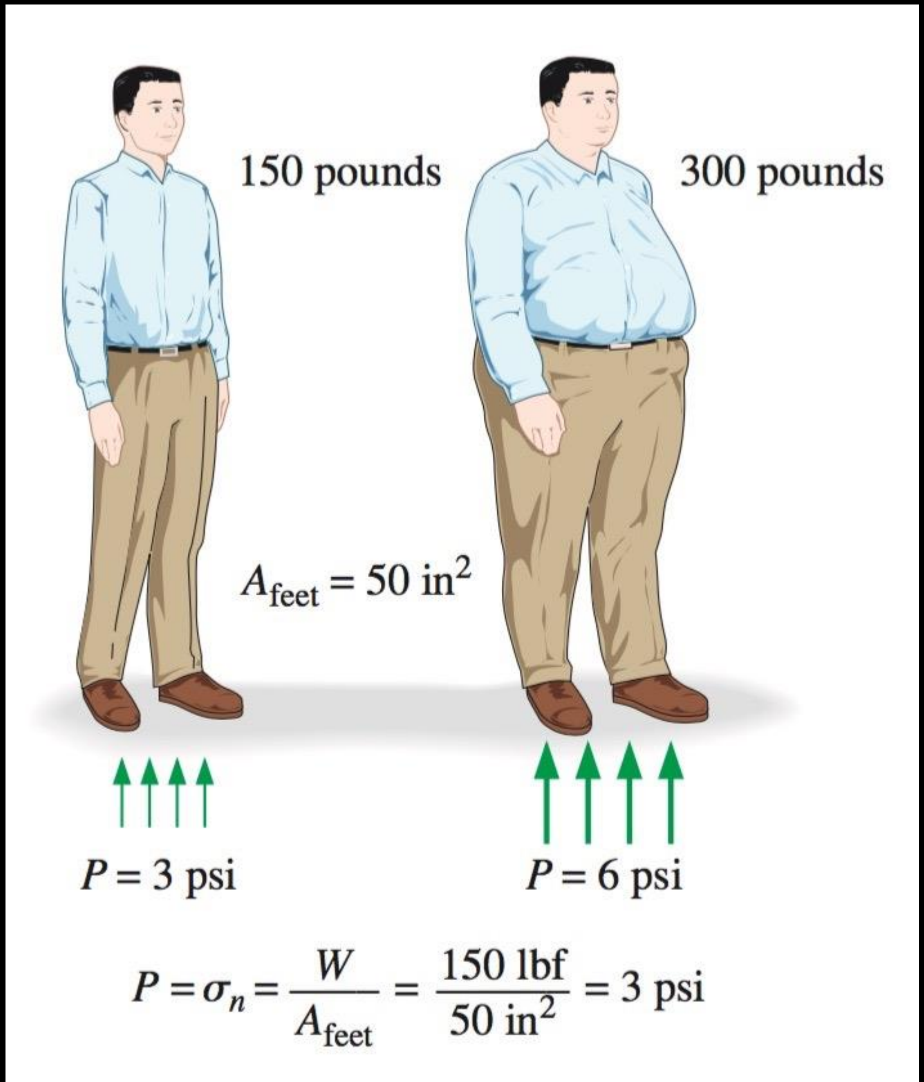
with a grant from the National Science Foundation

2 Statics of Fluids: Hydrostatics

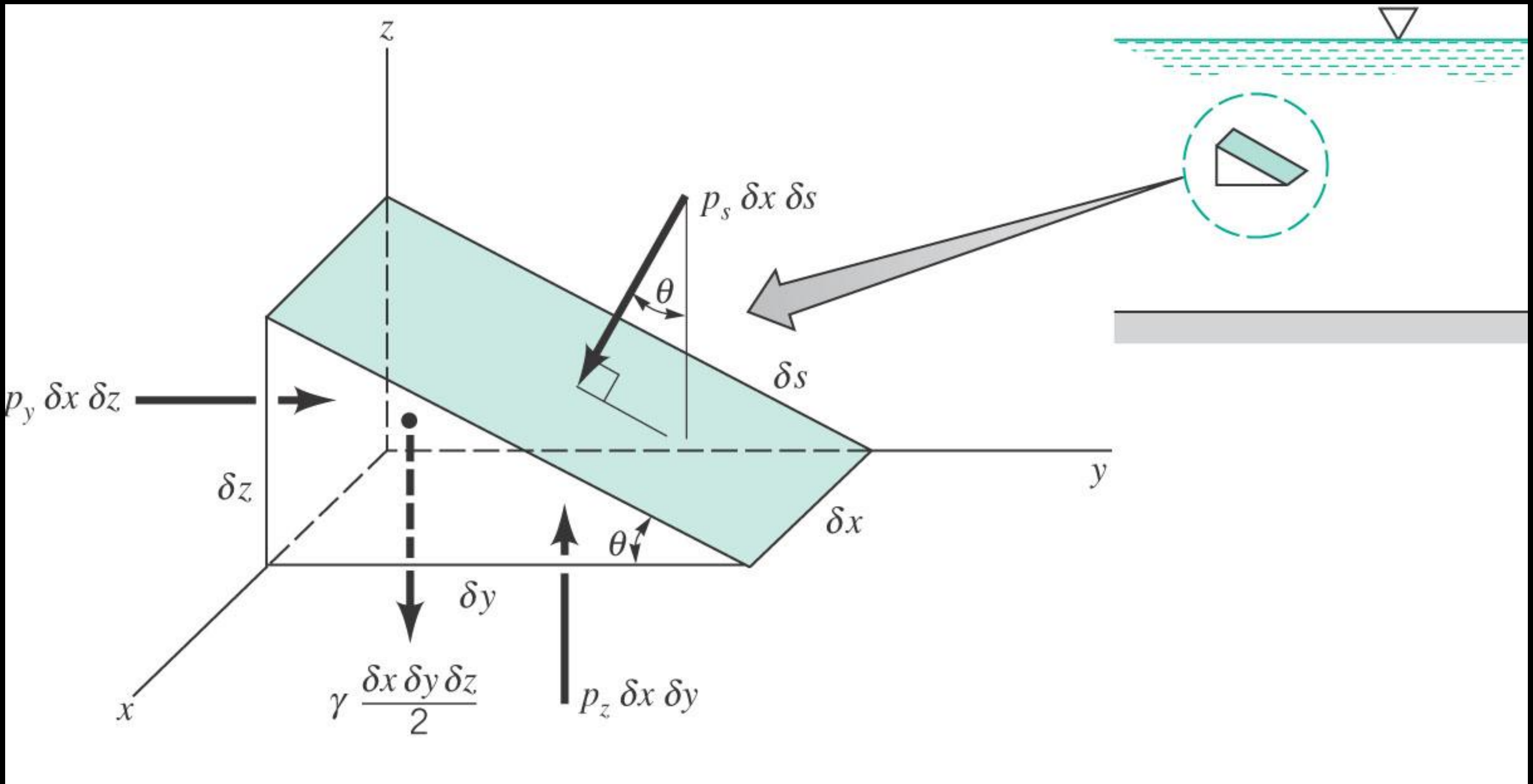
Pressure *is defined as a* *normal force exerted by a fluid per unit area.*

it has the unit of newtons per square meter ***(N/m2)****, which is called* ***a pascal (Pa)***

We speak of pressure only when we deal with a gas or a liquid. The counterpart of pressure in solids is normal stress.



A triangular prism and forces acting on it



$$p_y = p_s = p_z$$

$$\sum \vec{F} = m.\vec{a} = N\vec{F} + F\vec{F} + B\vec{F} = 0$$

as hydrostatic condition is being analyzed.

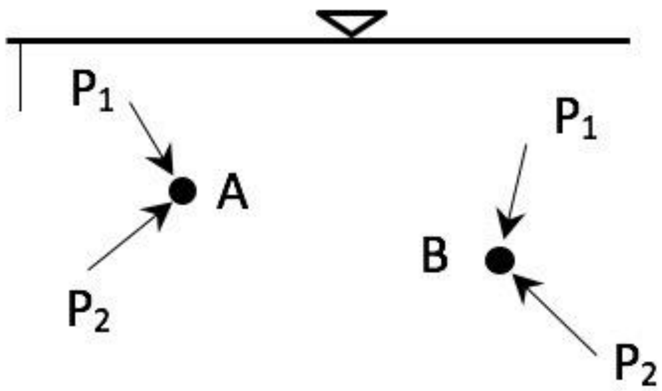
In addition, as there is no motion that can cause frictional force.

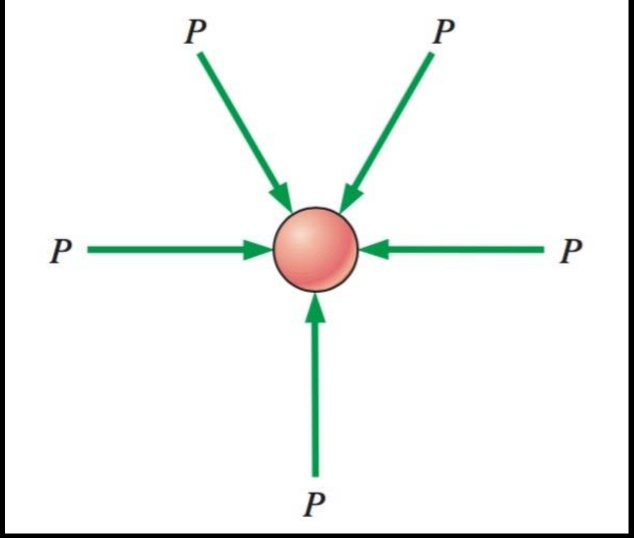
Therefore, we are only left with normal force and volumetric (body) force.

$$N\vec{F} + B\vec{F} = 0$$

Note:

1. $P_x = P_y = P_z = P$. It is not a function of direction making it scalar
2. This scalar pressure is the normal stress acting perpendicular to a plane
3. $P \neq P(dirction)$, but $P = P(\text{point}) = P(x, y, z)$





Pressure is a *scalar* quantity, not a vector; the pressure at a point in a fluid is the same in all directions.

2.2 The change in pressure along depth

The law of Hydrostatic pressure

We earlier saw that $P = P(\text{point}) = P(x, y, z)$. But, in reality, the most encountered condition is:

$P = P(z)$, and $P \neq P(x, y)$. We will use this principle to analyze the change in hydrostatic pressure with depth.

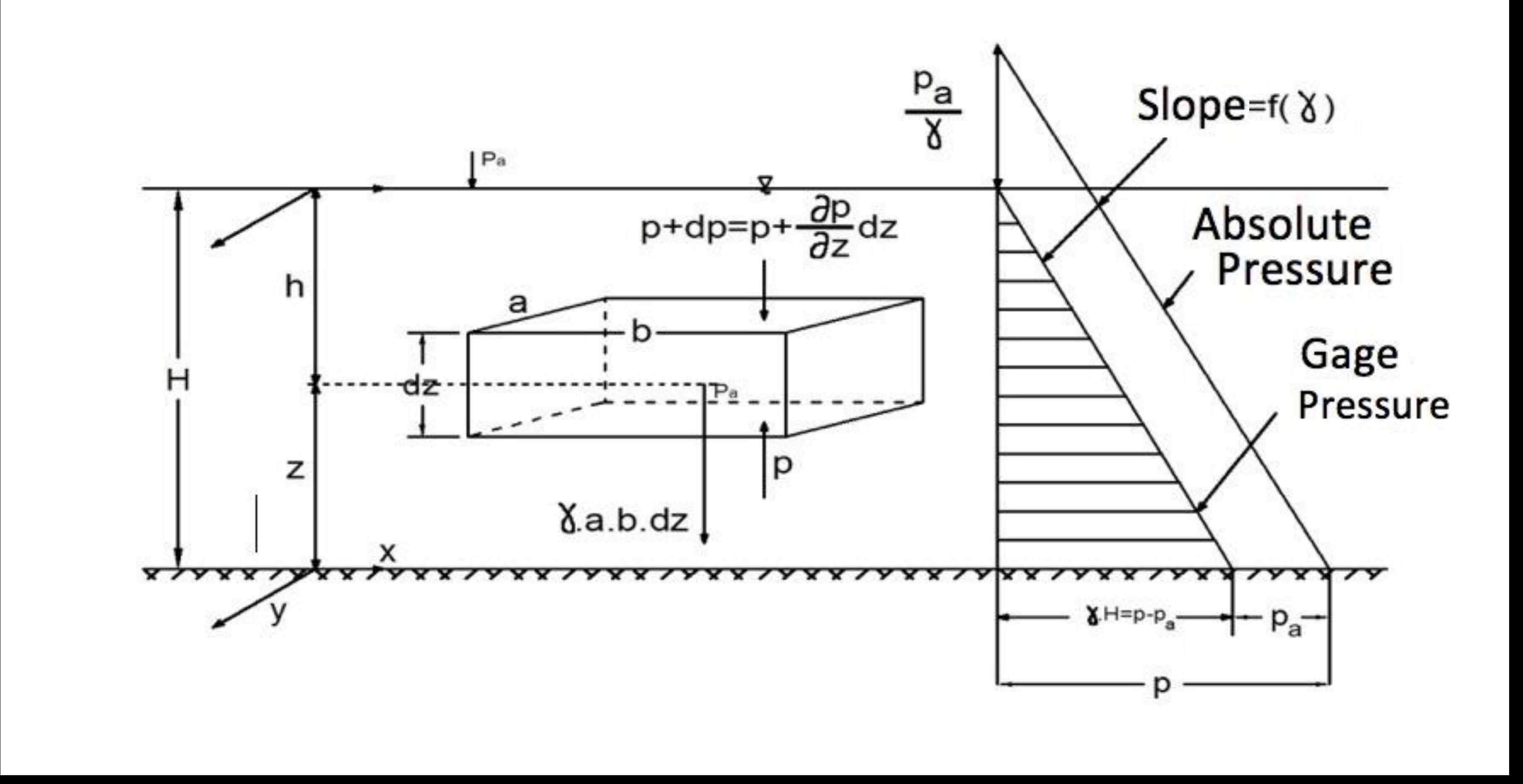


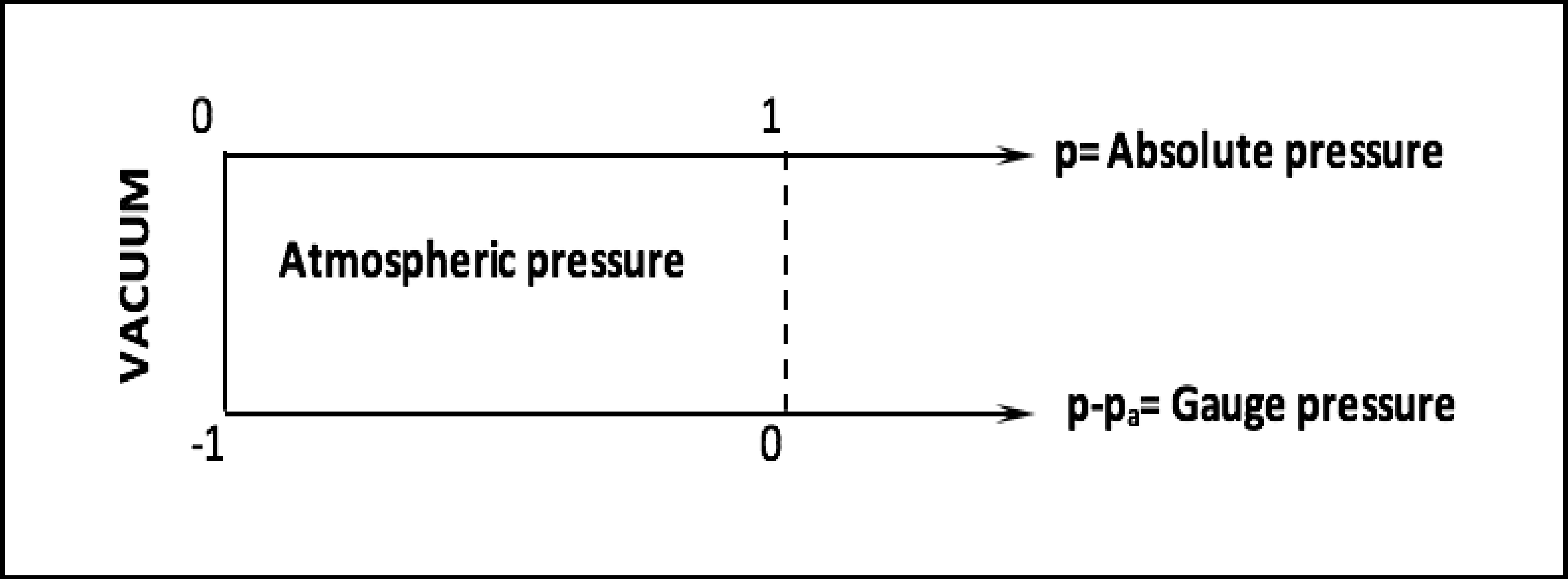
Fig.21: a cubic prism and pressures acting on it

Note:

- Pressure increases with depth.

- The pressure, P, is termed as **absolute pressure**. If we ignore the atmospheric pressure, we will only be remained with.

- The remaining pressure is called **gauge pressure**. Therefore, **gauge pressure** is the difference between the absolute pressure and atmospheric pressure. In practical applications, gauge pressure is generally used.



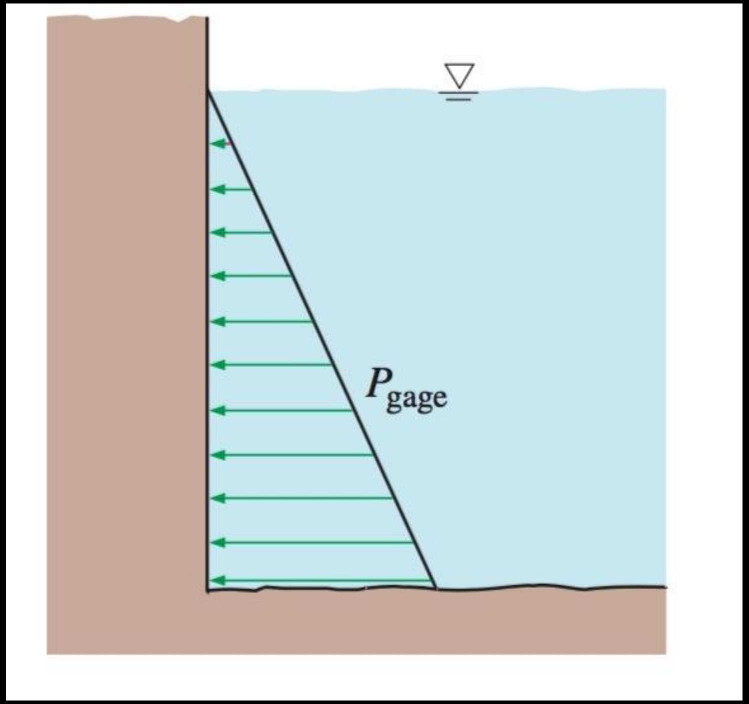
- The equation of hydrostatic pressure shows that pressure, P, increases linearly with depth, h.

- The surfaces where the pressure remains constant is termed as Neo-pressure surface.

If the specific weight is also constant, and since P is constant on neo-pressure surfaces, the depth, h, remains constant. This shows that points at the same depth are under the same hydrostatic pressure.

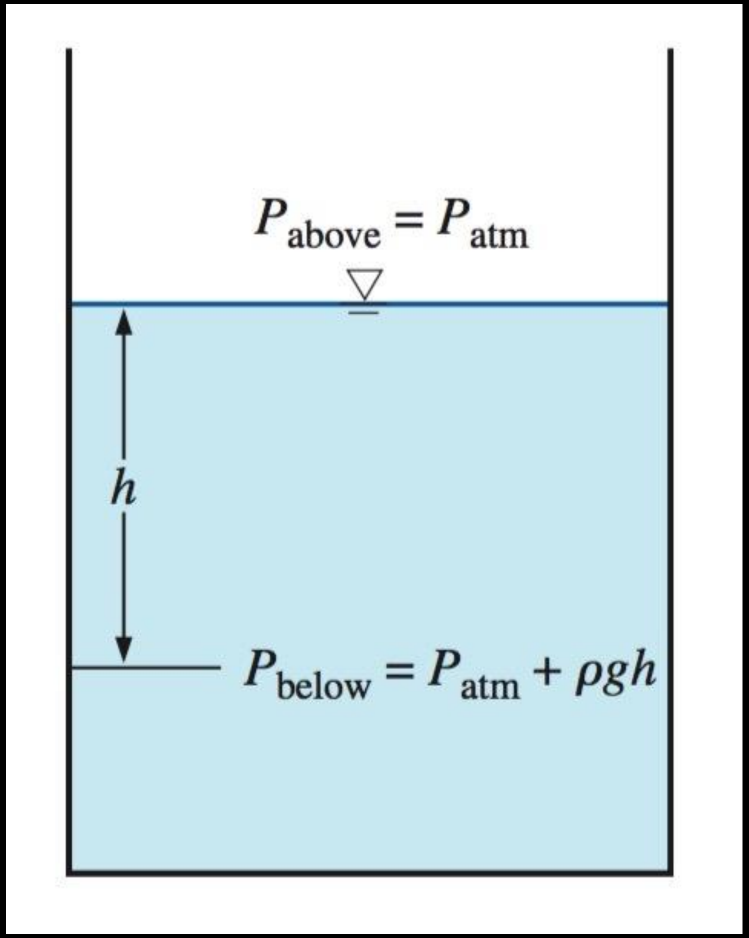
For a fluid at rest a = 0

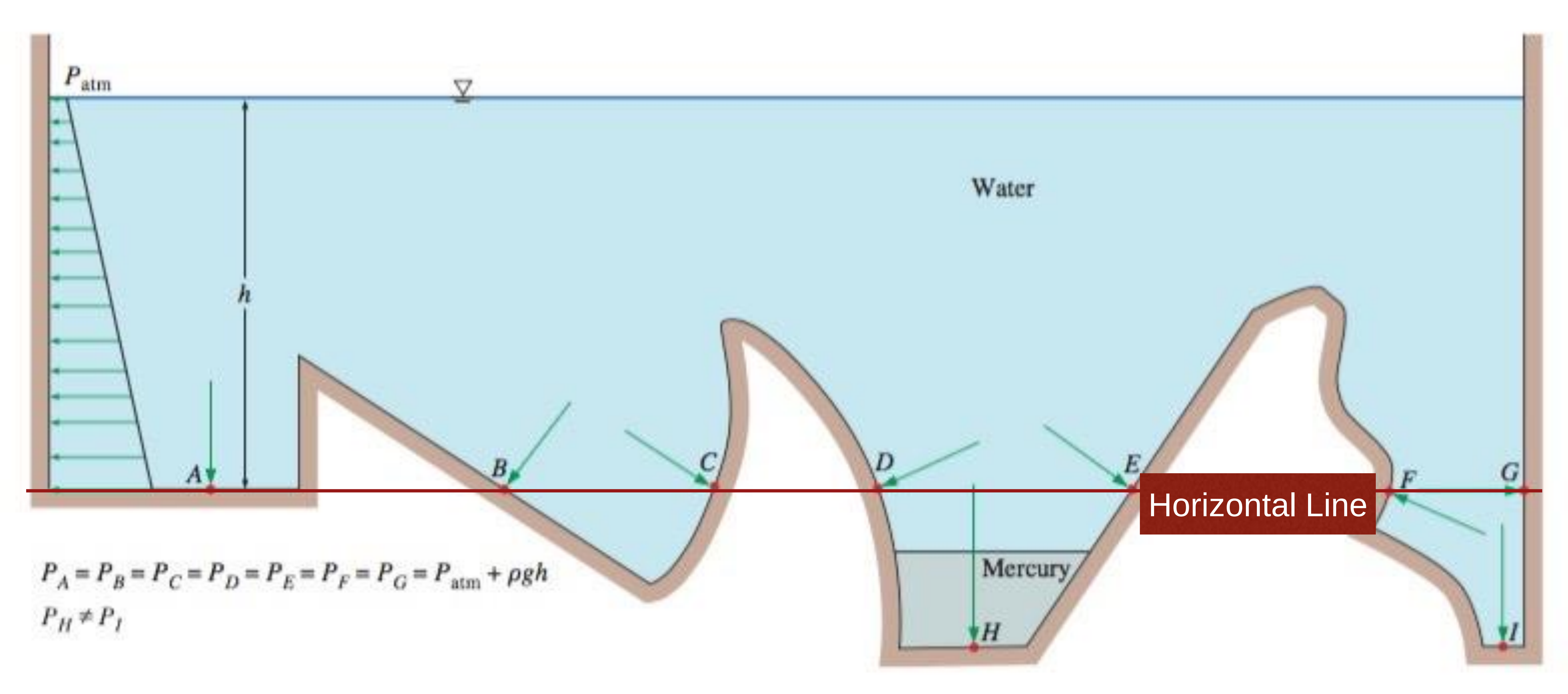
$$\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial z} = -\gamma$$



$$p = p_{atm} + \gamma h$$

(hydrostatic pressure distribution)



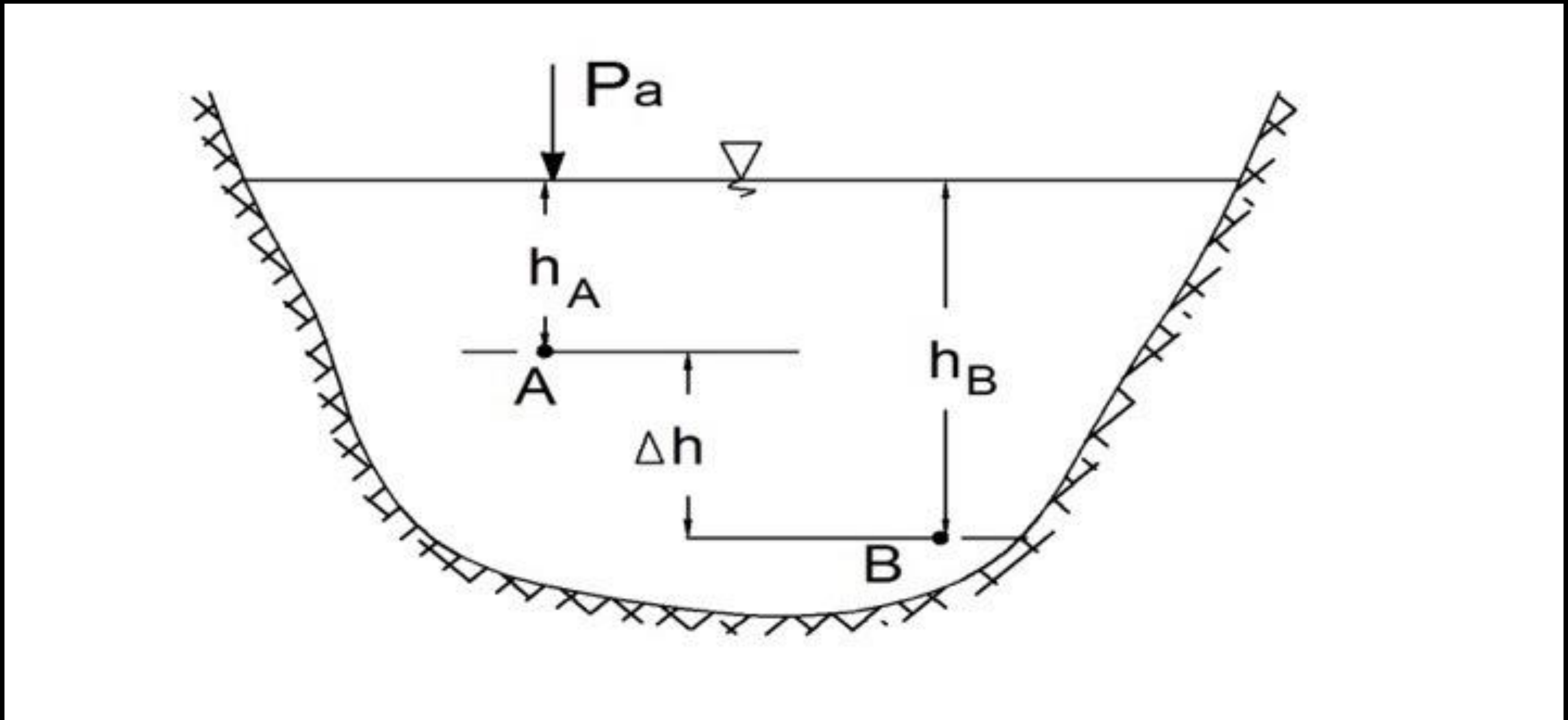


Under hydrostatic conditions, the pressure is the same at all points on a horizontal plane in a given fluid **regardless of geometry**, provided that the points are interconnected by the same fluid.

2.3 Practical applications of the law of hydrostatic pressure

2.3.1 Pressure difference between two points

Considering the given figure, we can write the hydrostatic pressure equation as point A and B.



A cross-section depicting points A and B and their depths from the surface

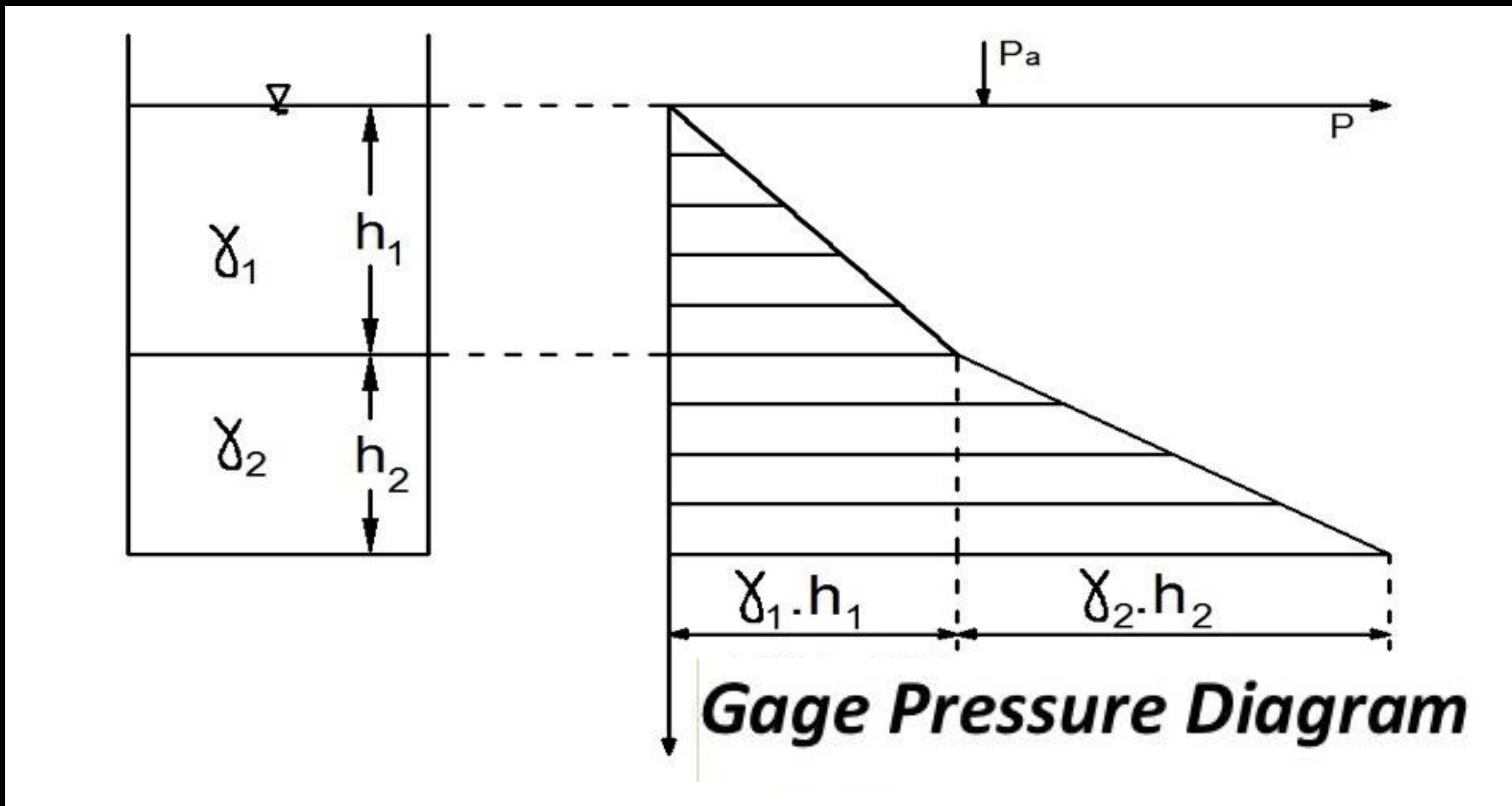
$$P_B = P_A + \gamma.\Delta h$$

$$P_B - P_A = \gamma(h_B - h_A)$$

The equation of hydrostatic pressure used to give pressure in relation with the depth of fluid.

The above relations makes the determination of pressure at any point possible if we know the pressure at another point.

Let’s consider to liquids which do not mix having specific weights and . The figure shows the relative pressure diagram for the two liquids.



Layers of two fluids one up on the other

The hydrostatic force on a surface submerged in a multilayered fluid can be determined by considering parts of the surface in different fluids as different surfaces.