Fluid Mechanics

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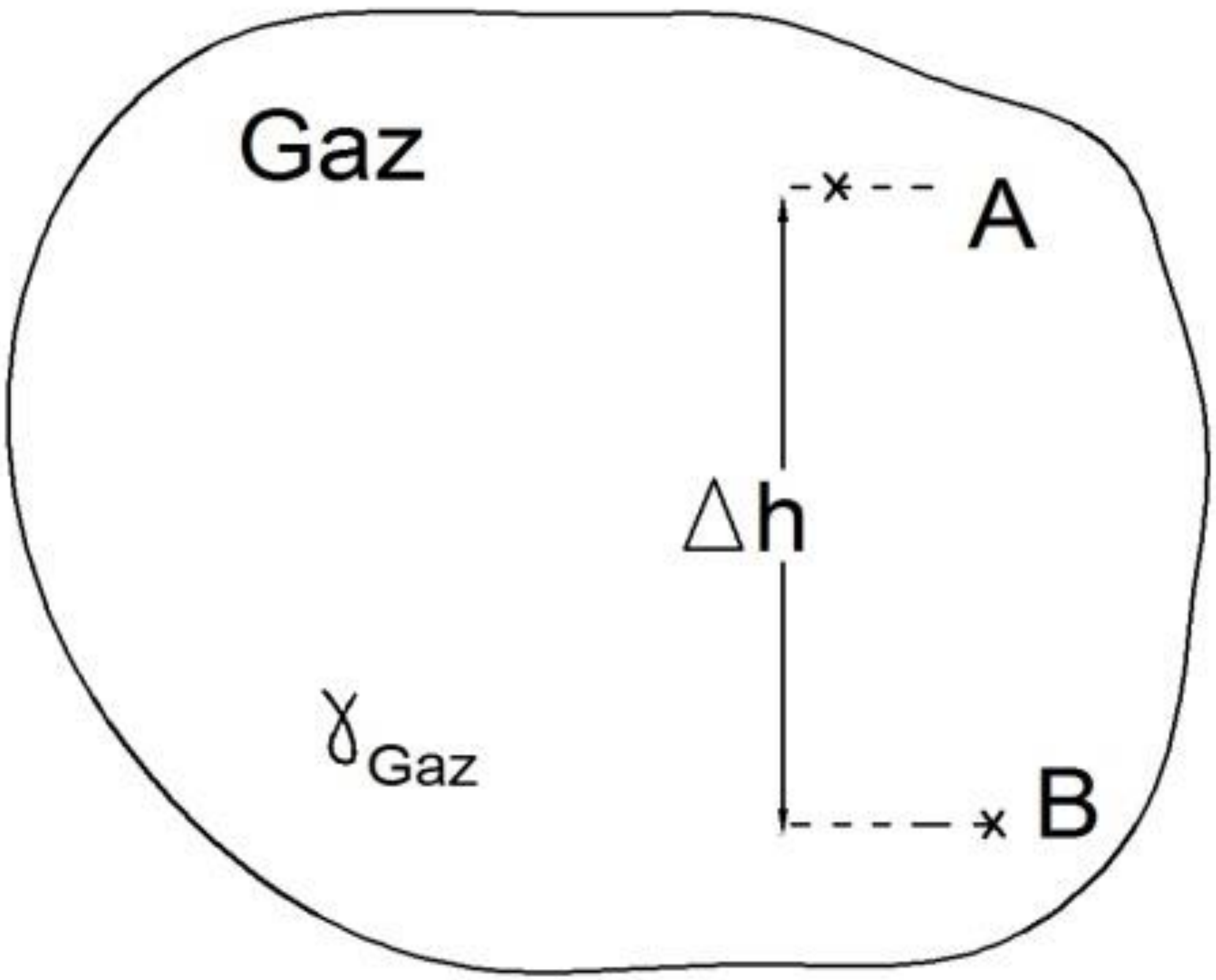
2.3.3 Pascal's Law

Let's consider a closed container filled with gas. We have seen earlier that the pressure at a point can be determined in relation with the pressure at another point. Accordingly, considering the figure:



a closed container filled with gas

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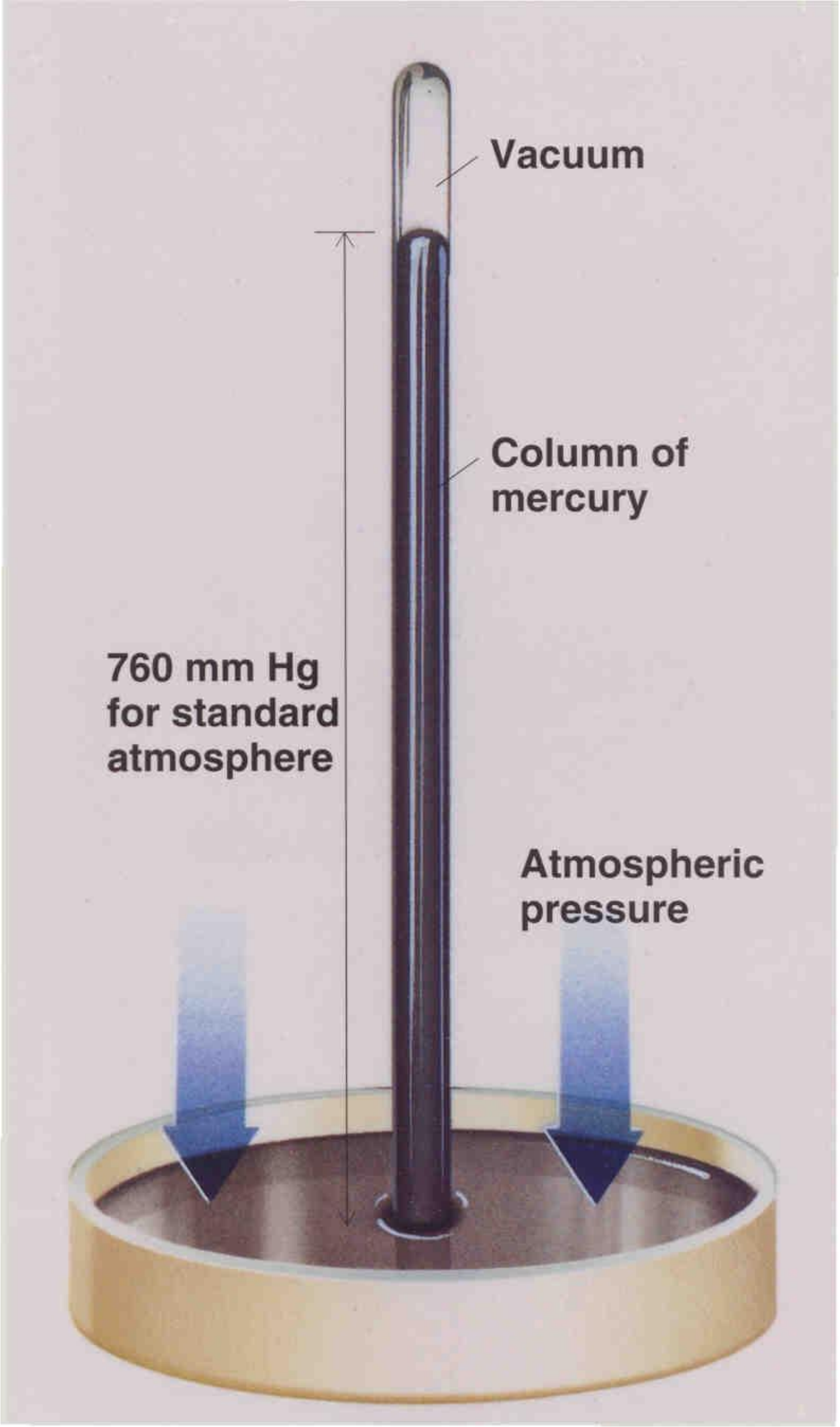
$$P_B = P_A + \gamma_{gas} . \Delta h$$

In a closed container filled with air or gas, the pressure at every point is the same. In order for the pressure at points A and B to be different, should be very big.

2.3.4 Torricelli's experiment

WE KNOW

The pressure at different points along the same depth with in the same liquid is the same.



Mercury barometer

$$P_{mercury} = P_{atm}$$

$$h_{mercury} = 0.76\ m$$

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Water Barometer

If we consider water in place of mercury,

$$P_{water} = P_{atm}$$

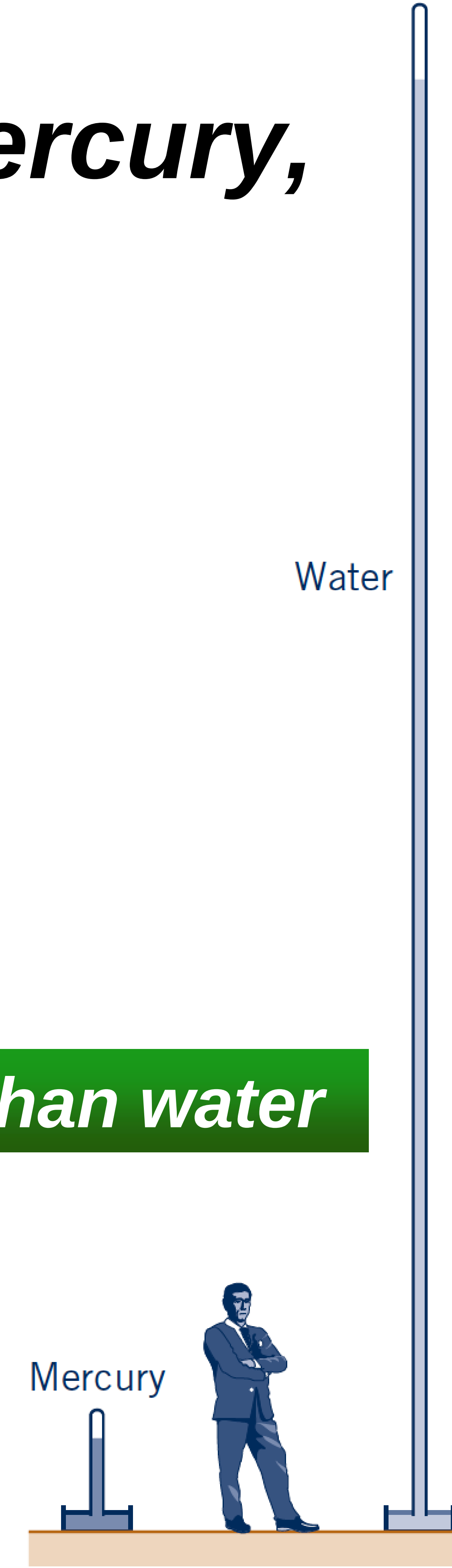
$$h_{water} = 10.33\ m$$

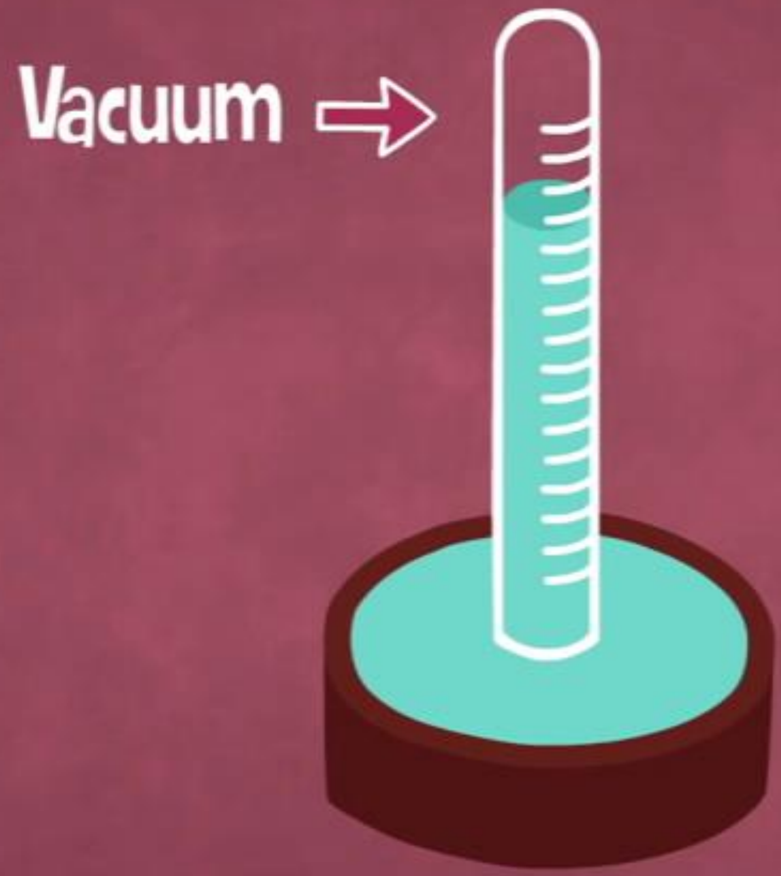
<u>This is why the pressure as a result of 10 m water depth taken as equivalent to atmospheric pressure.</u>

Specific Weight of mercury is 13.6 times bigger than water

Result:

- *<u>One atmospheric pressure is equivalent to the pressure that 10 m water column produces. This is called 1 atmosphere.</u>*



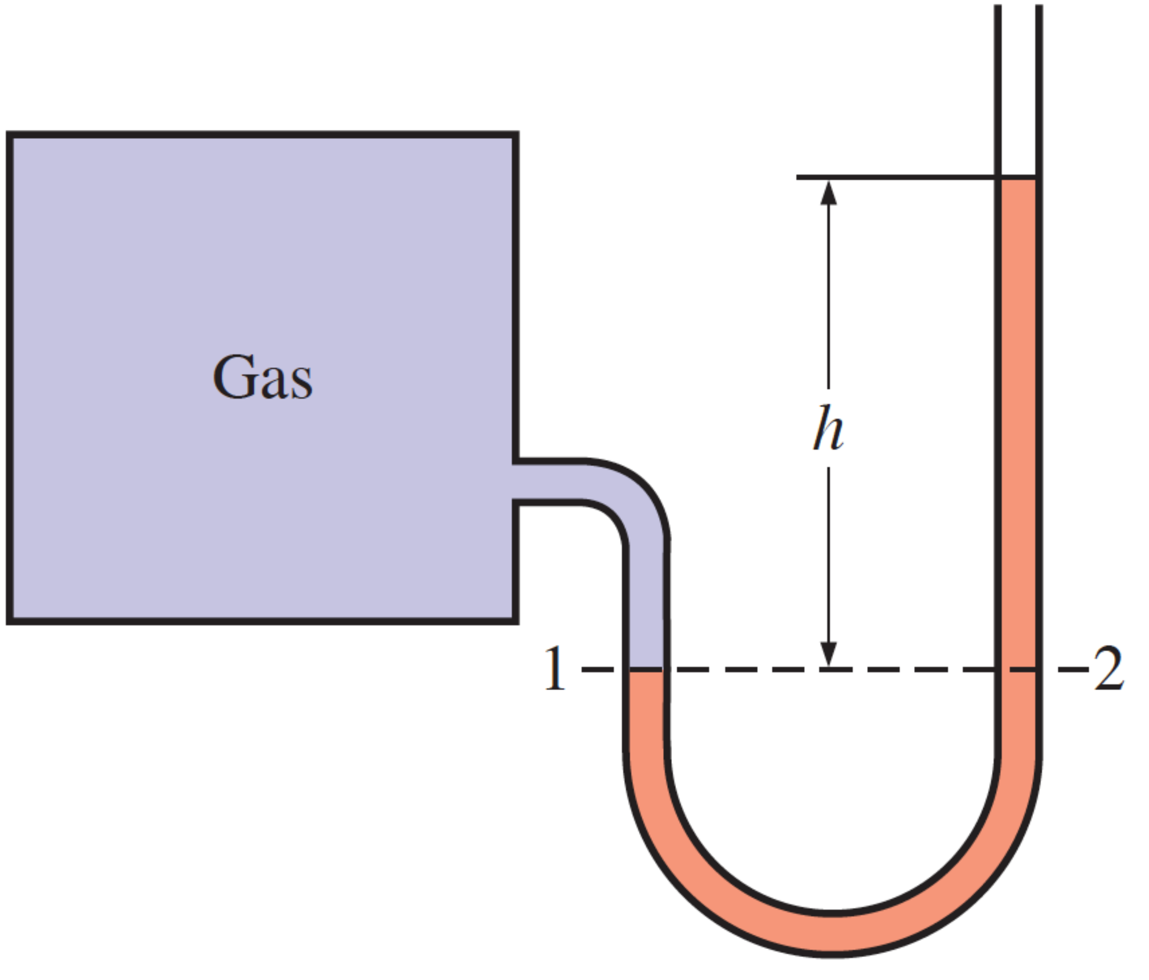

Vacuum

museo
galileo

2.3.5 Manometer

Manometer is an apparatus that uses column of liquid for measuring pressure.

The basic manometer.



A simple U-tube manometer, with high pressure applied to the right side.

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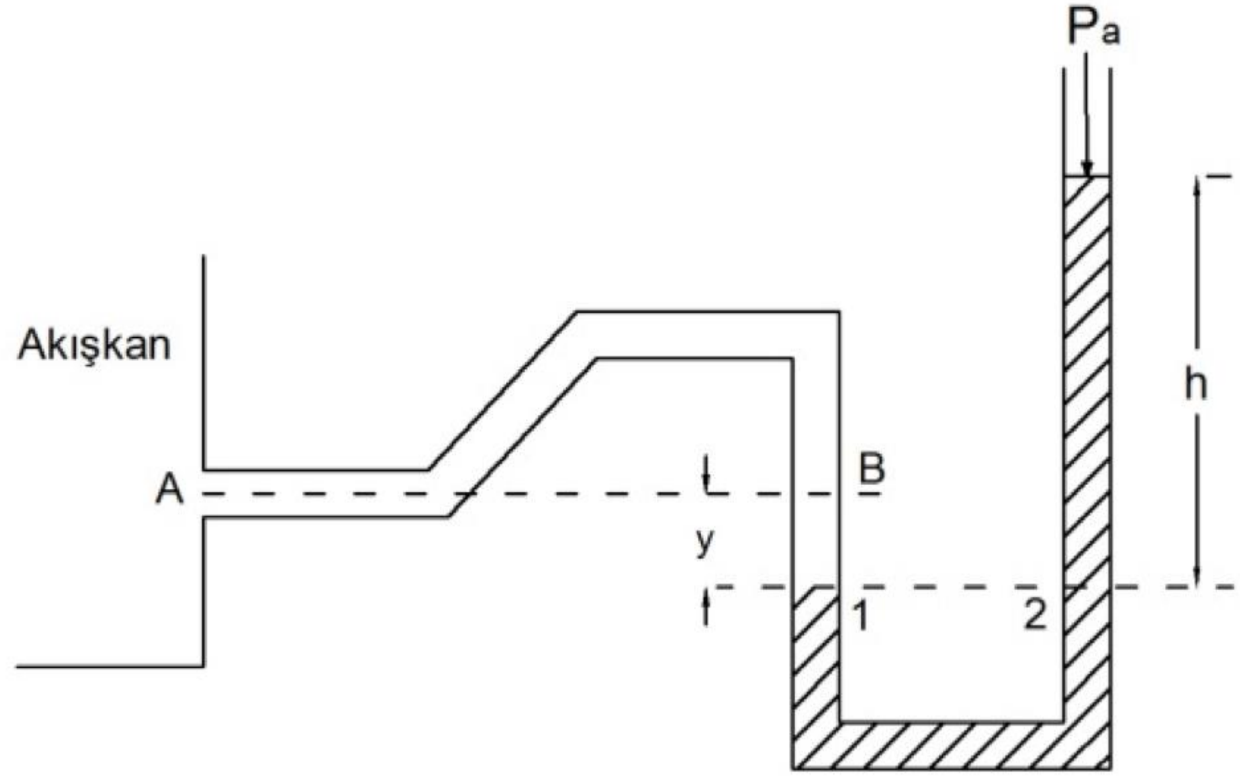

Therefore,

$$P_1 = P_2$$

$$P_1 = P_B + \gamma_{liquid} . y \qquad P_2 = P_a + \gamma_{mercury} . h$$

$$P_B = P_A = P_a + \gamma_{mercury} . h - \gamma_{liquid} . y$$

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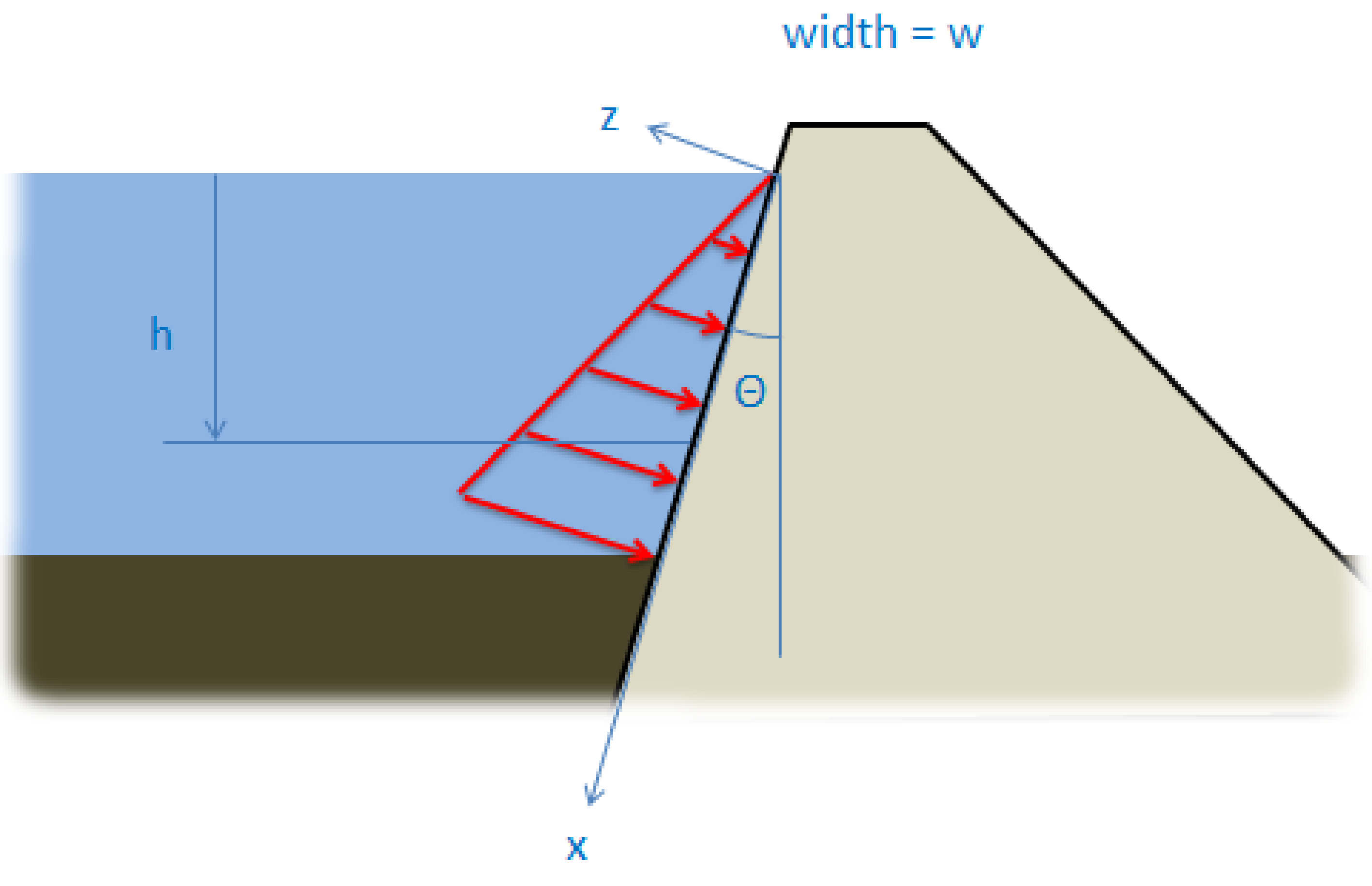
2.4 Calculation of Pressure Forces

The objective of Engineers is to determine the pressure force acting on the surface as a result of pressure acting on a surface.

***Two cases** could be considered in dealing with pressure forces.*

Case 1: Elemental Pressure forces acting planar surfaces.



*In this case, the elemental pressure forces are parallel to each other. **As a result, the resultant force can be calculated directly.***

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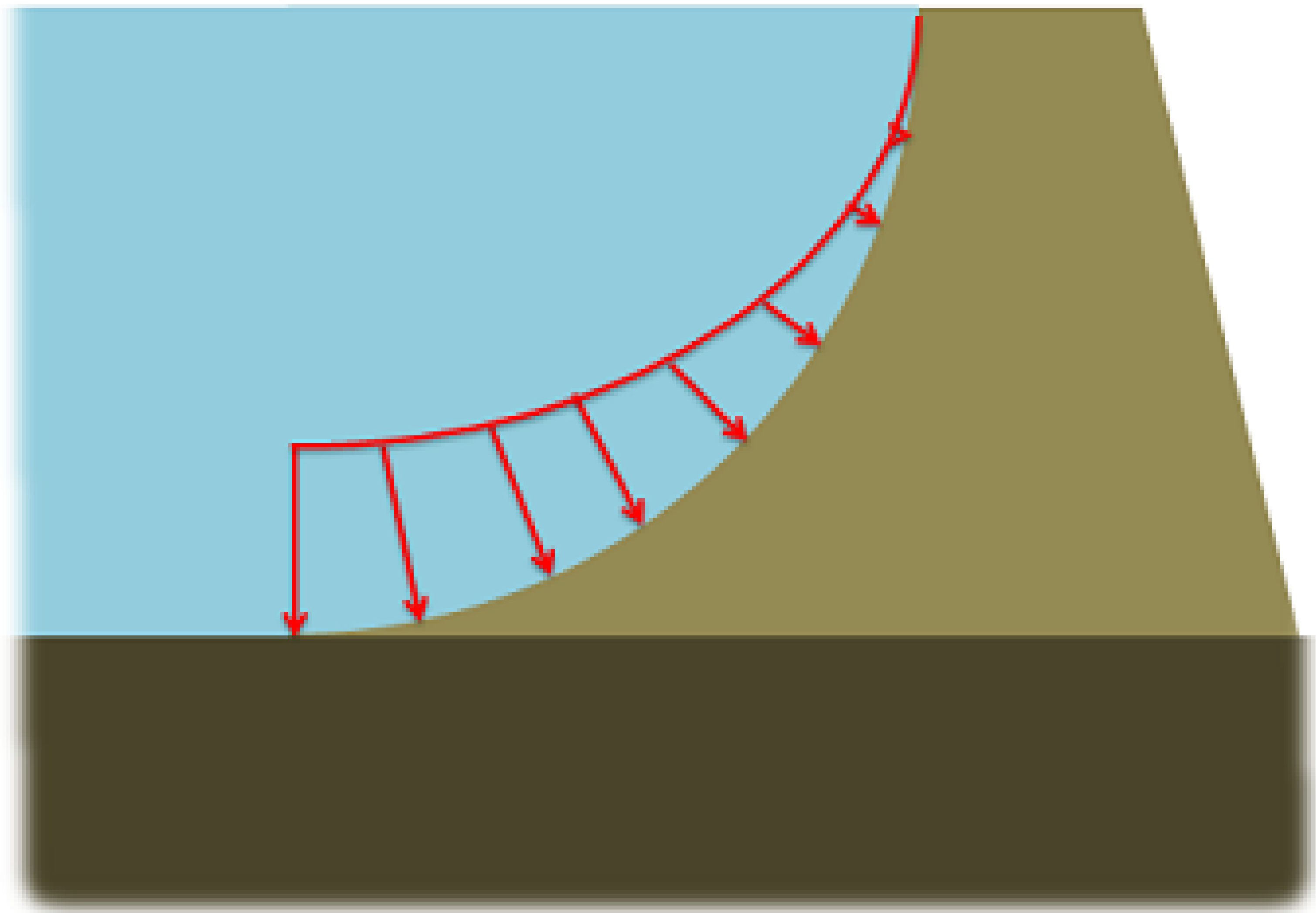
Case 2: Elemental pressure forces acting on curved surfaces.



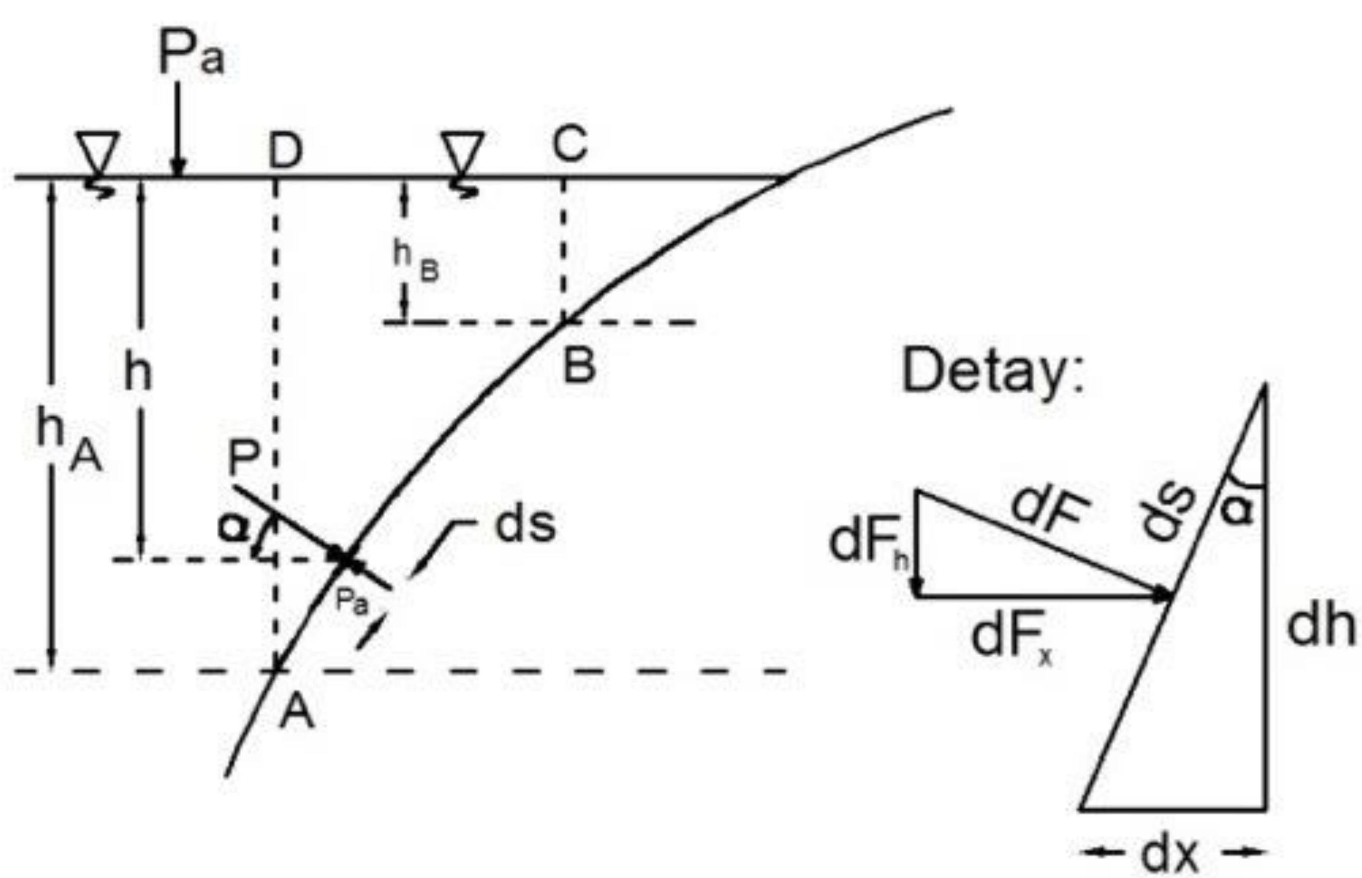
In this case, however, the elemental forces are not parallel to each other. ***As a result, the total pressure force cannot be calculated directly. It is rather calculated with the help of components.***

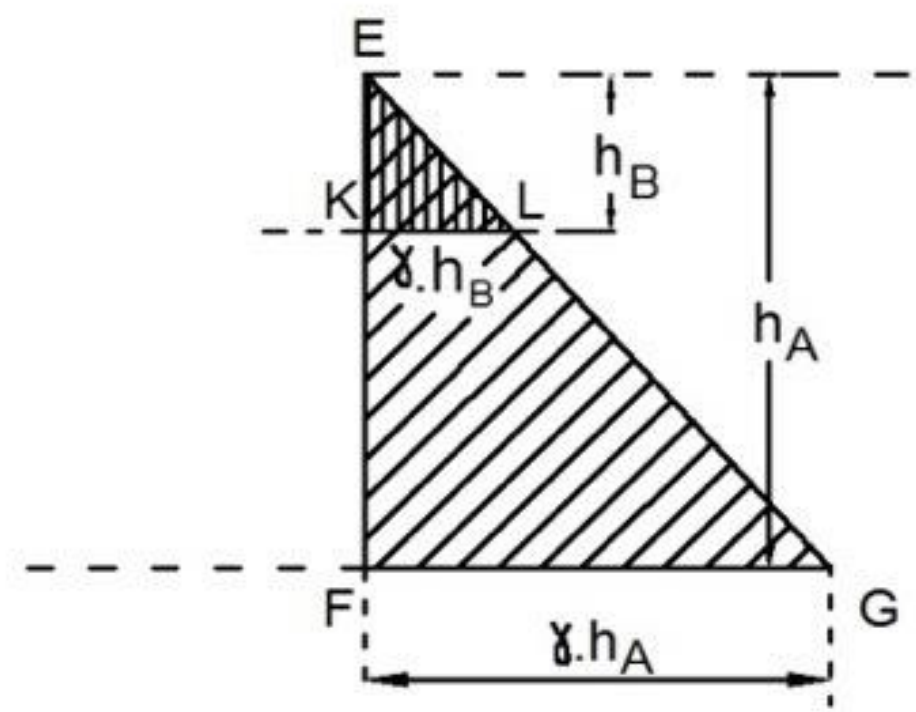
Here under, the general method for calculating pressure forces is given.

The first case (forces acting on planar surface) will be examined as a special case of the general method.

2.4.1 Pressure forces acting on cylindrical surfaces

For simplification, let’s consider 2 dimensional plane, a curved surface in one direction and the dimension perpendicular to the figure plane is B.





The cylindrical surface and forces acting on it

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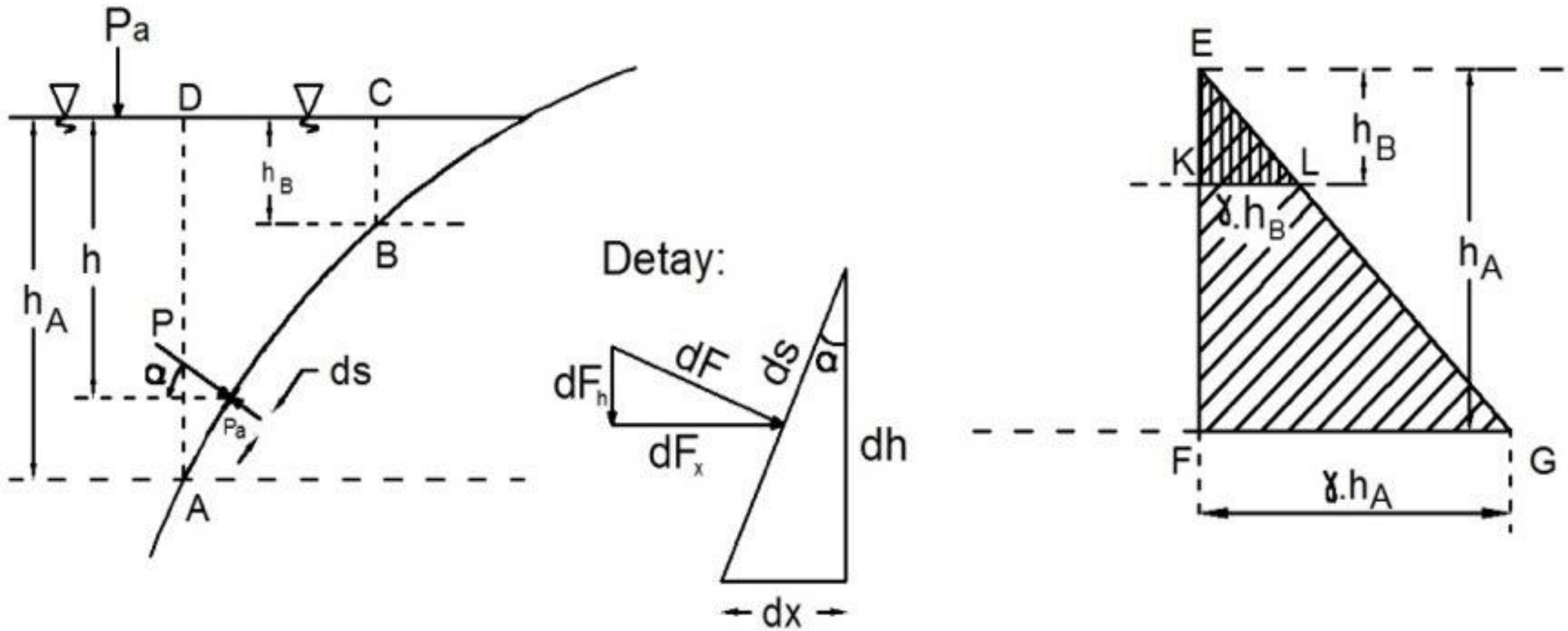

$$F_x = b.\gamma.\frac{\left(h_A^2 - h_B^2\right)}{2} \qquad \text{and} \qquad F_h = b.\gamma.(DABCDarea)$$

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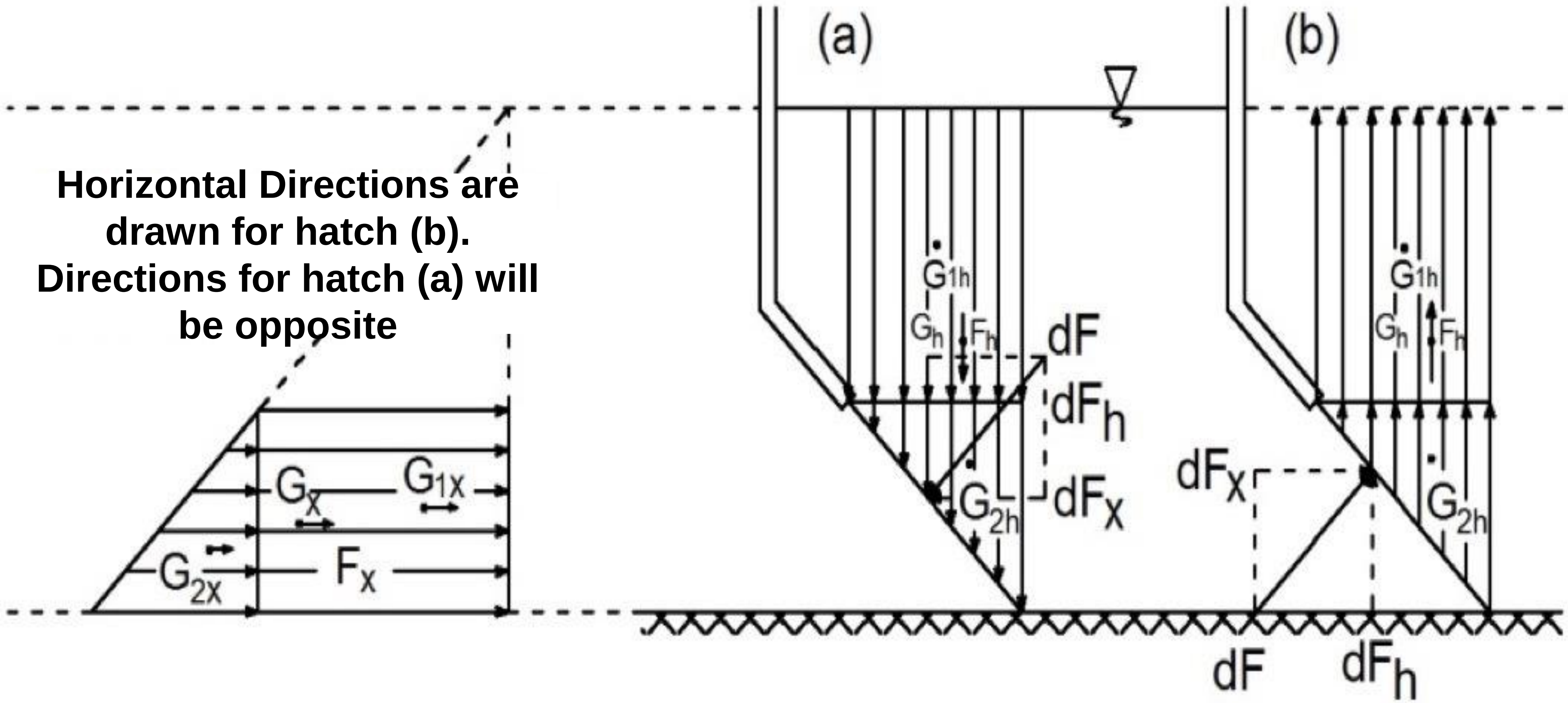
Notes

1. In order to avoid error when determining the directions of forces Fx and Fh, the use of elemental forces is beneficial.

Horizontal Directions are drawn for hatch (b). Directions for hatch (a) will be opposite



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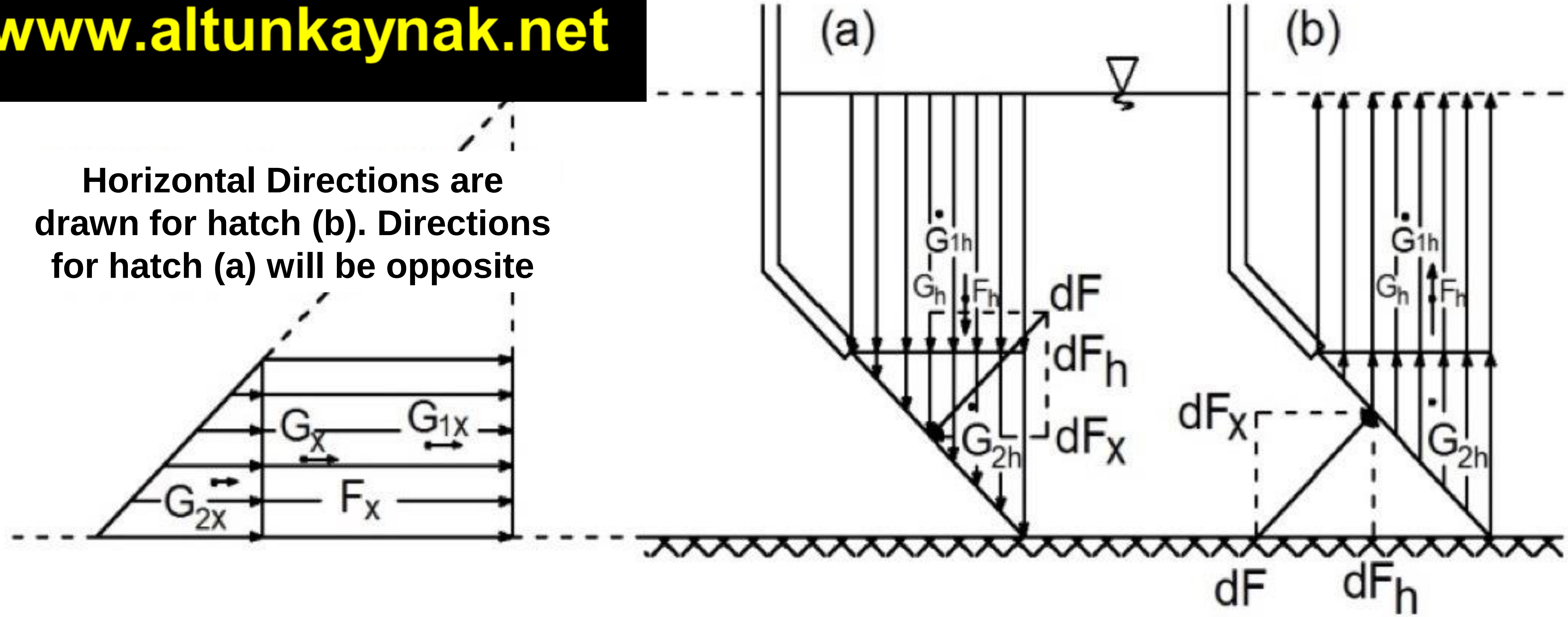
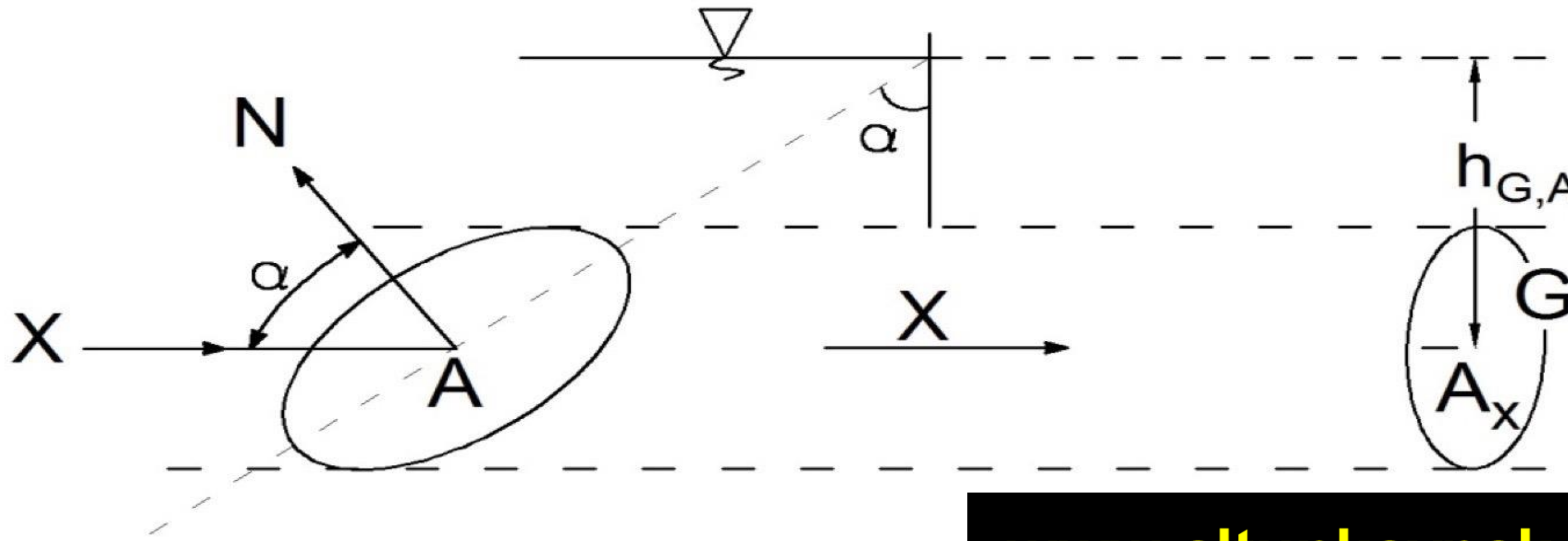


2. When determining the points at which Fx and Fh are acting, one should take in to consideration the center of gravities of the areas used in the calculation of the forces.

G1: center of gravities of rectangular areas
G2: center of gravities of triangular areas
G: center of gravities resulting from rectangle + triangle. These are points where Fx and Fh components are passing through.

3. Earlier, we considered a 2-d cylindrical surface for simplification. We can show the calculation of forces on non-prismatic (non-cylindrical) surfaces using the same logic. (We will use 3-d non-cylindrical surface when driving Archimedes' principle).

4. If ''dA=b.ds'' is used in the equations given above, the equations will have the following forms.



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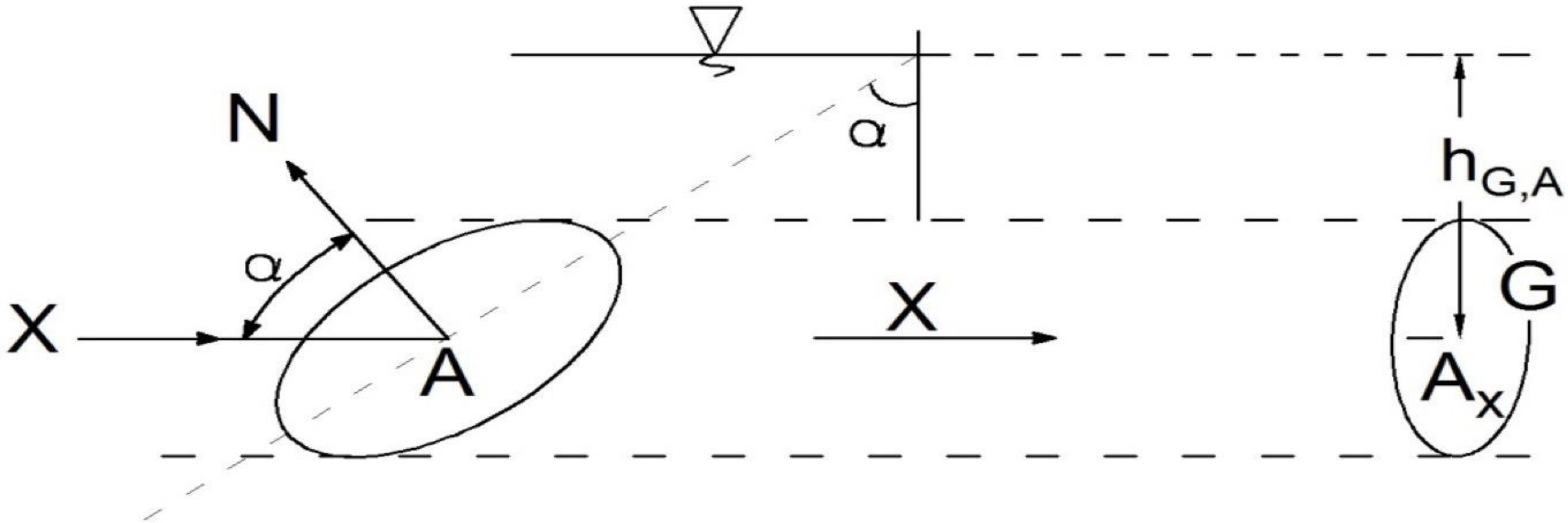

$$dF_x = \gamma . h . dA_x \text{ and } dF_h = \gamma . h . dA_h$$

If static moment is considered, the equation will take the following forms:

$$F_x = \gamma . \int_{A_x} h . dA_x = \gamma . h_{G,A_x} . A_x \text{ and } F_h = \gamma . \int_{A_h} h . dA_h = \gamma . \forall$$

The h_{G,A_x} *and* A_x *in the first relationship show the depth of the center of gravity of the projected area.*

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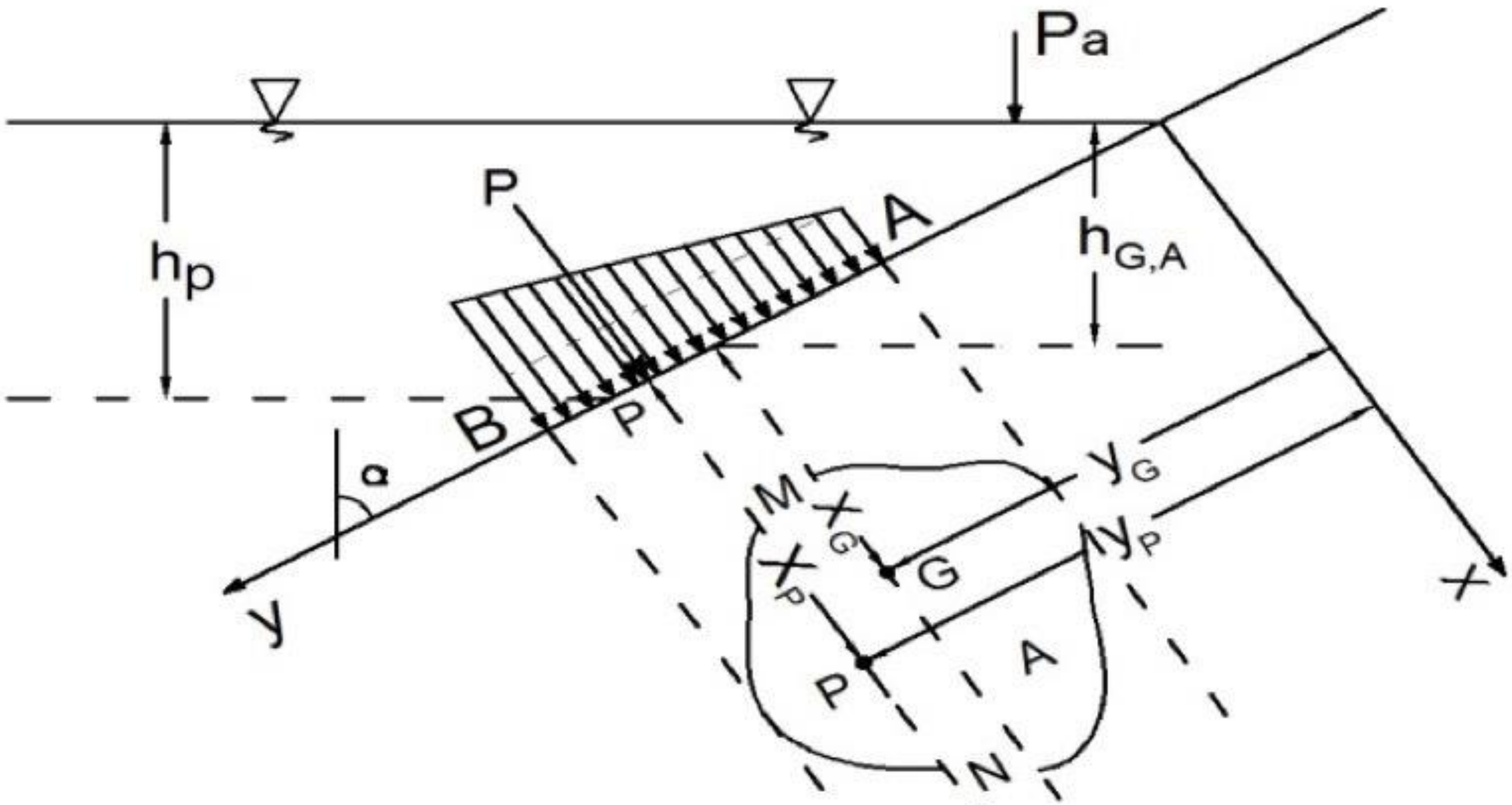
5.Special case: planar surface condition



In this special condition,

$$F = \int_A dF = \gamma . \int_A h.dA = \gamma . h_{G,A} . A$$

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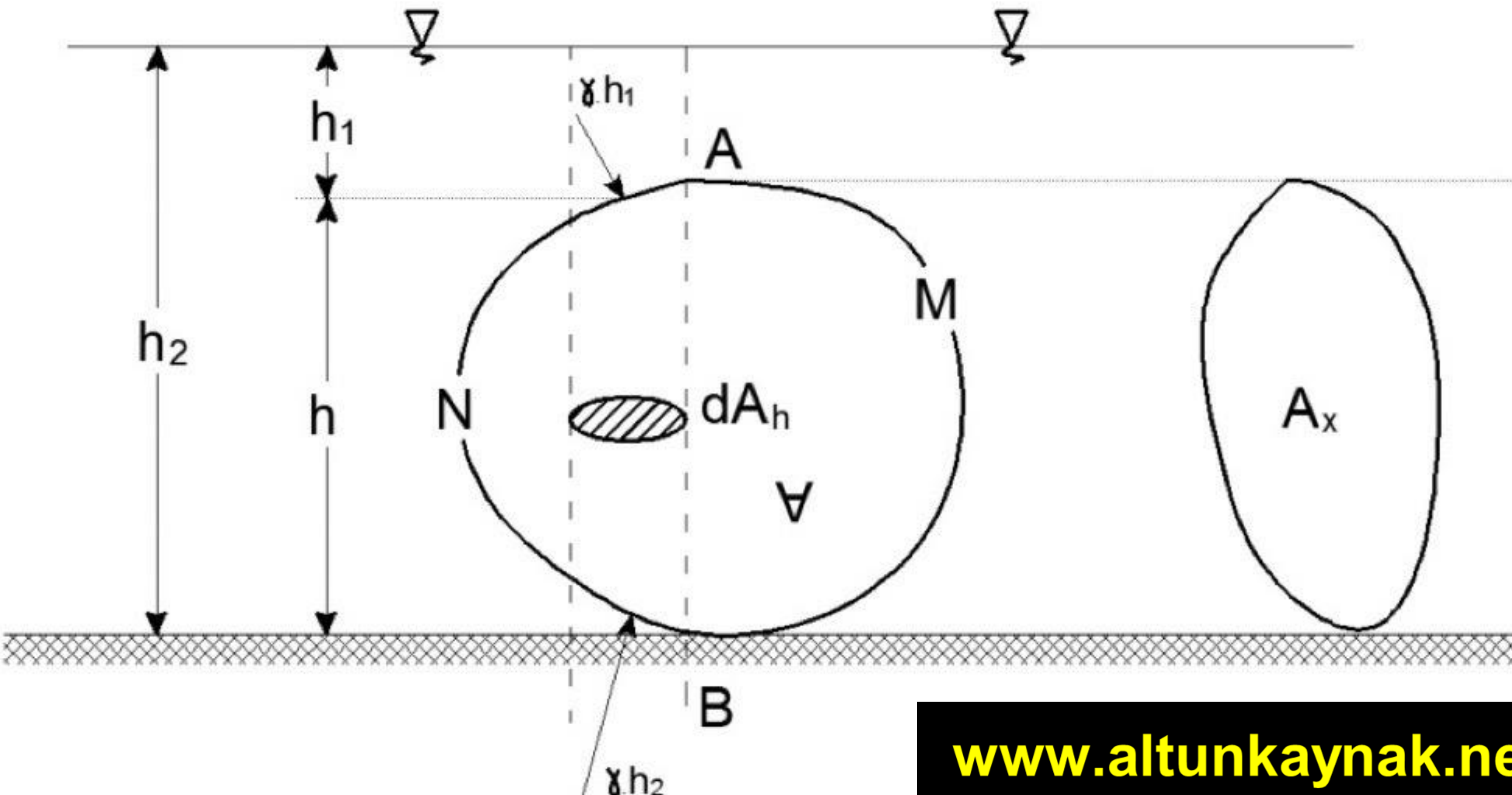
1.4.1Archimedes' Principle

This deals with the pressure distribution a body immersed in a fluid.

Let's have a body immersed in water as shown in the figure.



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The resultant force acting on area dA_h is: $dF_h = \gamma . d\forall$

This is the Archimedes principle and the force is the uplift force acting on an immersed body.

The total vertical force is: $F_h = \int_{\forall} \gamma . d\forall = \gamma . \forall = K$

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Note:

In Archimedes principle

Bouyant Force is

$$F_h = \gamma_{liquid} \cdot \forall_{body}$$

and

The weight of the body is

$$W = \gamma_{body} \cdot \forall_{body}$$

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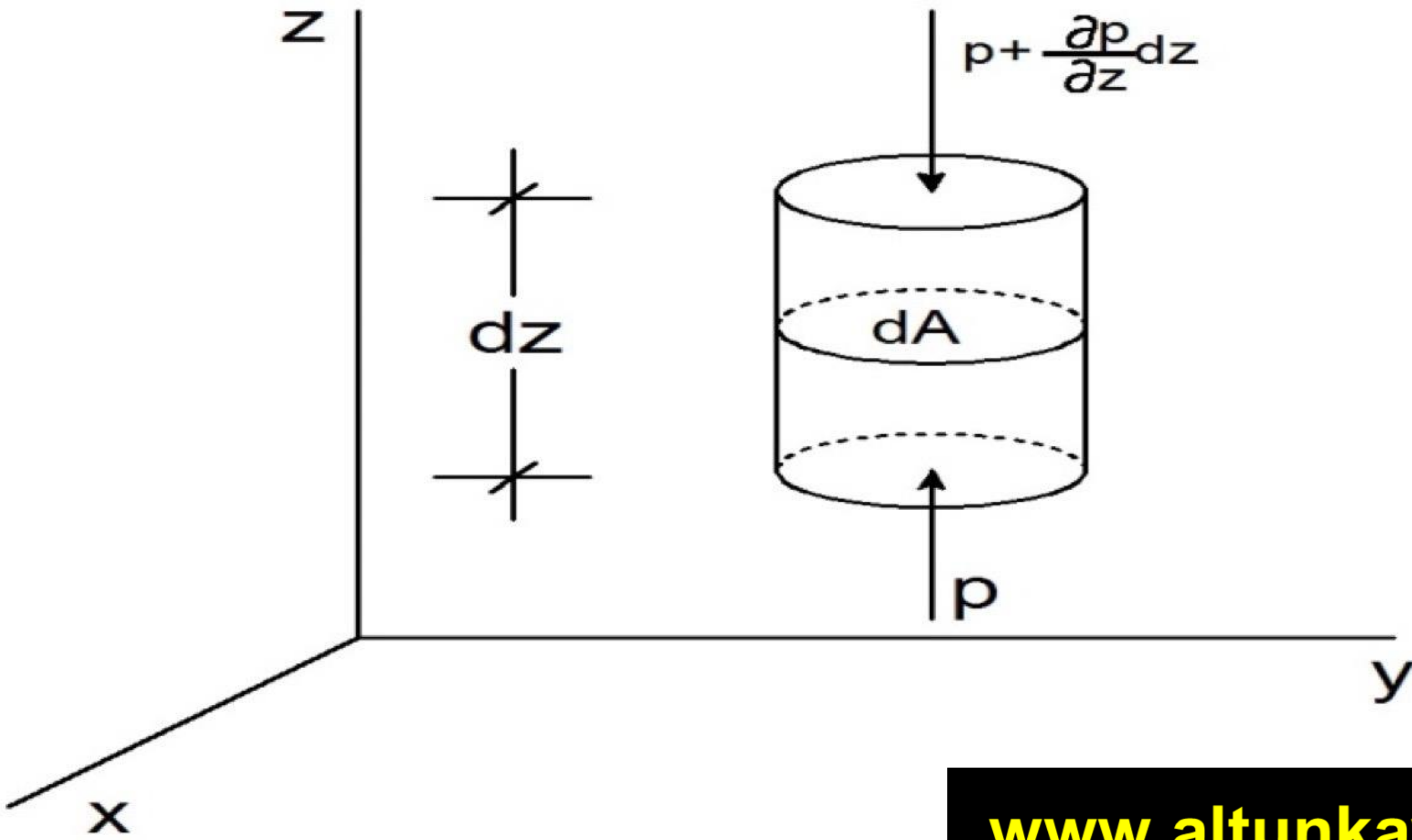
By pressing the sides of the bottle, the pressure within it is increased and the air within the inverted test tube is compressed. This allows additional water to enter the test tube, thereby causing the average specific weight of the object (the test tube, air, and the enclosed water) to be greater than that of the surrounding water. The tube sinks.

Clouds are often a result of buoyant effects in the atmosphere. As light weight, warm humid air rises it cools, and the moisture condenses as clouds.Conceptually, the rise of this hot air is similar to the rise of a ping-pong ball that is released underwater.

2.5 Generalization

Hydrostatic basic equations:

Euler's Equilibrium Equations:



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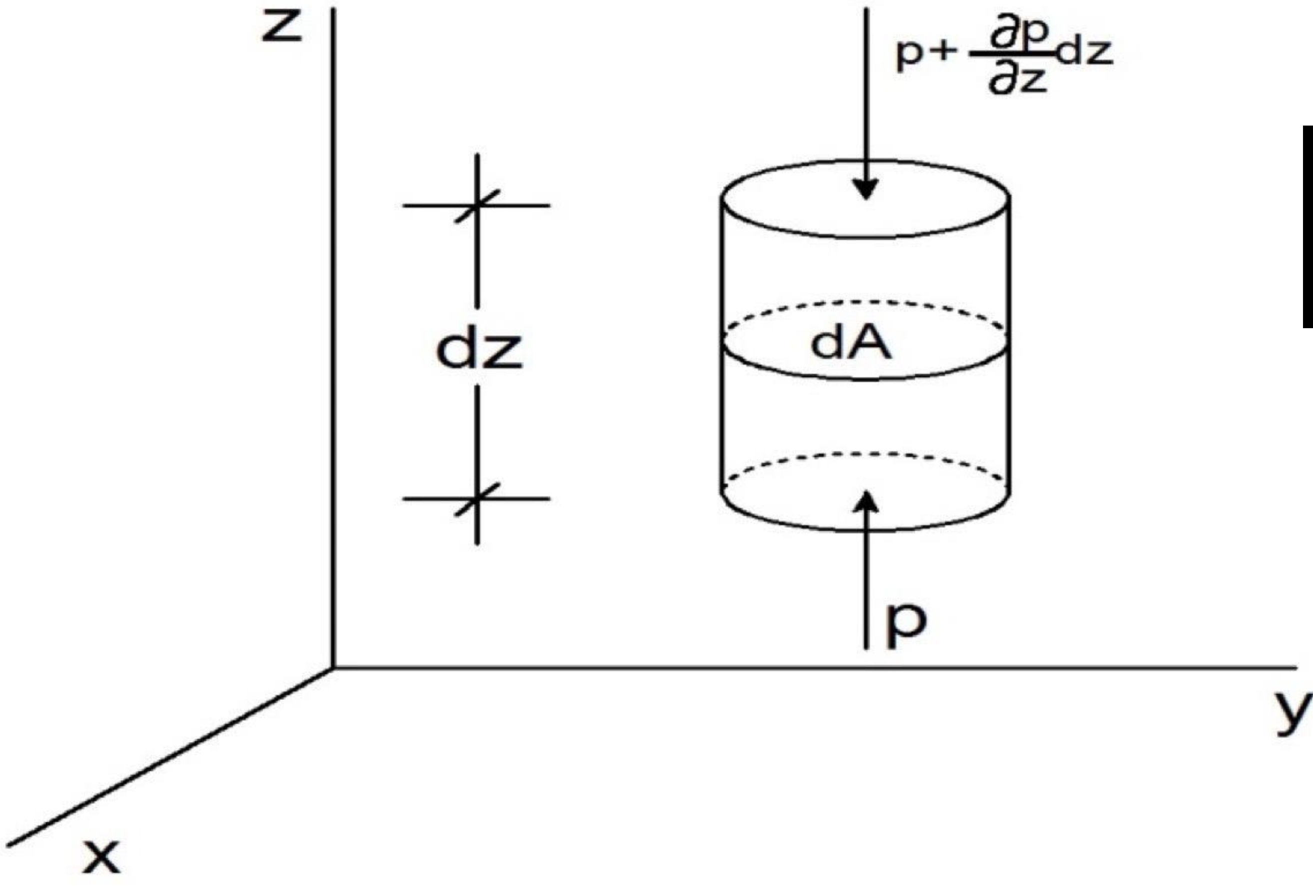

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If we write the volumetric force (k) in terms of its components, it will have the following form

$$K = K_x\vec{i} + K_y\vec{j} + K_z\vec{k} = (K_x, K_y, K_z)$$

Newton's law of motion

$$\sum \vec{F} = m.\vec{a} = N\vec{F} + F\vec{F} + B\vec{F}$$

$F\vec{F}$ *is zero as velocity is zero, which implies acceleration is also zero as the flow is under hydrostatic condition.*

Therefore, taking summation of forces in the z-direction:

$$\sum \vec{F}_z = m.\vec{a}_z = 0$$

$$\frac{\partial P}{\partial z} = \rho.K_z$$

In the same manner

$$\frac{\partial P}{\partial x} = \rho.K_x \quad \text{and} \quad \frac{\partial P}{\partial y} = \rho.K_y$$

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2.5.1 Special condition

In conditions where the volumetric force acts

only in the vertical direction as a result of gravitational acceleration, g

$$K_x = K_y = 0 \text{ and } K_z = -g.$$

$$\frac{\partial P}{\partial x} = 0 \text{, but } P \neq P(x) \quad \text{and} \quad \frac{\partial P}{\partial y} = 0 \text{, but } P \neq P(y)$$

and, therefore, $P = P(z)$

This implies that $\dfrac{\partial P}{\partial z} = \dfrac{dP}{dz} = -\rho.g = -\gamma$

$$P = -\gamma.z + C$$

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Taking boundary conditions

at $z=H$, $P=P_{atm}$

$$P = P_a + \gamma . h$$

This is the hydrostatic pressure equation.

Note:

It should be noted here that the above equation was based on the fact that

$$P \neq P(x), P \neq P(y) \text{ and } P = P(z)$$

where the volumetric force acts vertically downward as a result of g.

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In its general form, we wrote this equation as

$$\frac{\partial P}{\partial s} = \rho.K_s$$

In terms of components, we have, $\frac{\partial P}{\partial x} = \rho.K_x$, $\frac{\partial P}{\partial y} = \rho.K_y$ and $\frac{\partial P}{\partial z} = \rho.K_z$

We can write K_x, K_y and K_z as a function of U=U(x,y,z).

$$K_x = -\frac{\partial U}{\partial x},\ K_y = -\frac{\partial U}{\partial y} \text{ and } K_z = -\frac{\partial U}{\partial z} \Rightarrow -\rho.dU = dP$$

In its general form, Euler's Equation can be written as follows.

$$P(x,y,z) + \rho.U(x,y,z) = C$$

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$$P(x,y,z)+\rho.U(x,y,z)=C$$

Accordingly,

1. The value of the function should remain the same at every points throughout the system
2. is the force potential function
3. This kind of motions are called Conservative motions
4. In terms of coordinates (s,n,b), the components of the force can be written as:

$$K_s=-\frac{\partial U}{\partial s},\quad K_n=-\frac{\partial U}{\partial n}\quad \text{and}\quad K_b=-\frac{\partial U}{\partial b}$$

Therefore, in its most general form, the equation can be written as:

$$P(s,n,b)+\rho.U(s,n,b)=0$$

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5. The velocity potential function is defined as

$$\phi = \phi(x, y, z)$$

and we know that

$$u = \frac{\partial \phi}{\partial x},\ v = \frac{\partial \phi}{\partial y} \text{ and } w = \frac{\partial \phi}{\partial z}$$

Considerations:

1. The surfaces where pressure is constant are called neo surfaces or iso-pressure surfaces. Similarly, the surfaces where U remains constant are termed as iso-potential surfaces.

Therefore, from , we can conclude that the iso-pressure surfaces are the same as the iso-potential surfaces.

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Considerations:

2. Special condition: considering gravitational acceleration

$$K_x = K_y = 0, \text{ and } K_z = -g \Rightarrow dP = -\rho g dz$$

Taking the integrals of both sides of this equation results in:

$$P = -\gamma Z + C$$

The value of the constant ‘C’ was dealt in the ‘Hydrostatic pressure’ section by taking boundary condition.

3. If we replace the force potential function in the above equations, we will get:

$$dP = -\rho\left(\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz\right)$$

$$\Rightarrow dP = \rho\left(K_x dx + K_y dy + K_z dz\right).$$

The change in iso-pressure on the surface is zero (dp=0). Accordingly,

$$dP = \rho\left(K_x dx + K_y dy + K_z dz\right) = 0$$

when written in its vector form

$$\Rightarrow \left(K_x dx + K_y dy + K_z dz\right) = \vec{K}.d\vec{s} = 0$$

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Results :

1. If $\vec{K}.d\vec{s} = 0$ is written in $\vec{K}.d\vec{s} = 0 = |\vec{K}|.|d\vec{s}|.\cos(\vec{K}, d\vec{s})$ form, it implies that $\vec{K}$ is perpendicular to $d\vec{s}$. Therefore, it can be concluded that volumetric fore is perpendicular to iso-pressure surfaces.

Special case:

Let volumetric force be only 'g'.

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$\Rightarrow g$ is vertical. Therefore, the iso-pressure surface should be horizontal.

It should be noted that the free surface is the iso-pressure surface because every point on this the free surface under the effect of atmospheric pressure.

Therefore, under gravitational field, the free surfaces of liquids is horizontal.

2. $\vec{K}.d\vec{s} = 0$ implies that force times distance is equal to zero. Therefore, the work that the volumetric force does along the neo surfaces zero as $\vec{K}$ is perpendicular to $d\vec{s}$.

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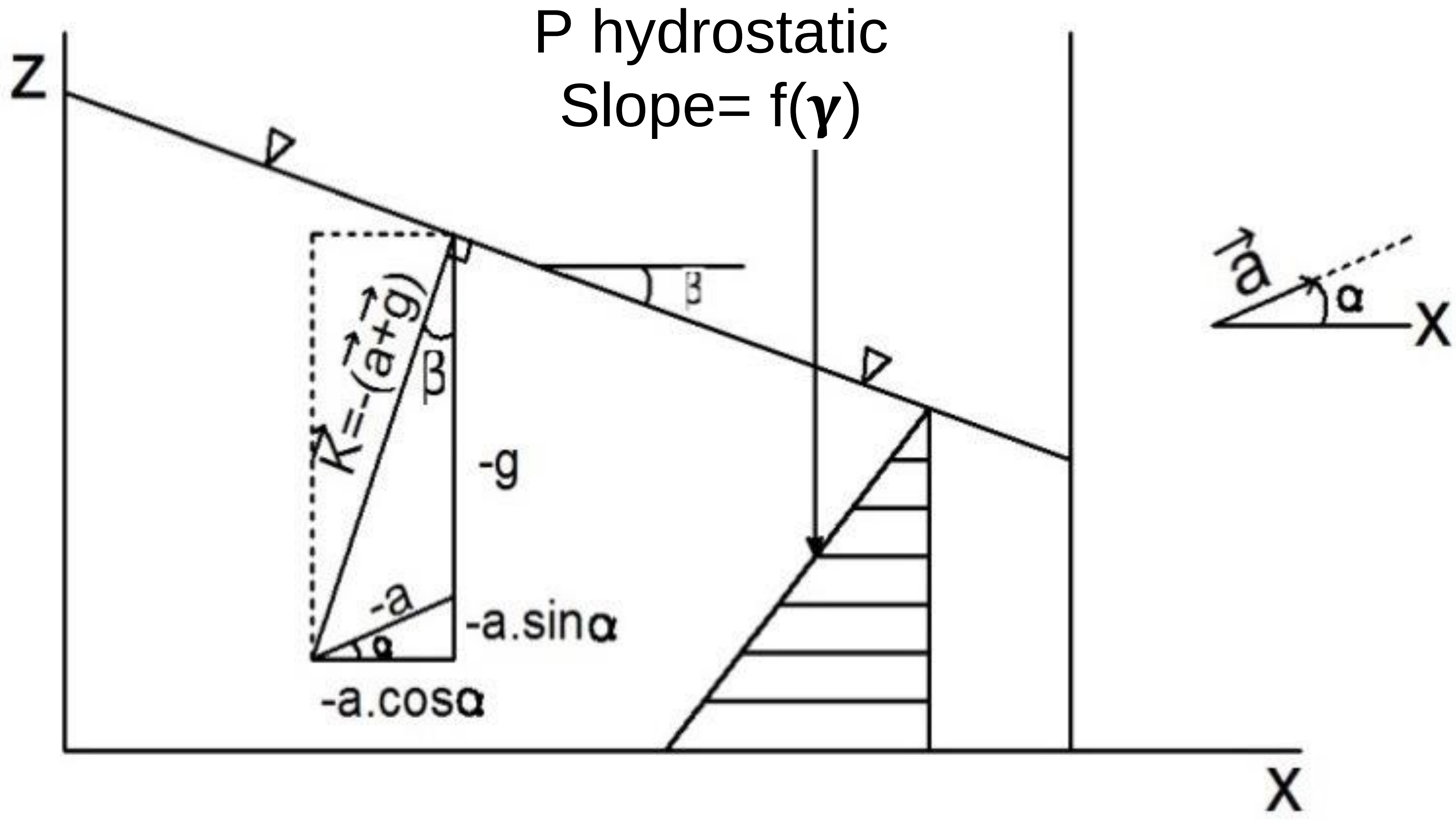
2.5.3 Applications: Liquids in relative equilibrium (Liquids that move like solids)

1. A container moving with constant acceleration:



A container moving with constant acceleration

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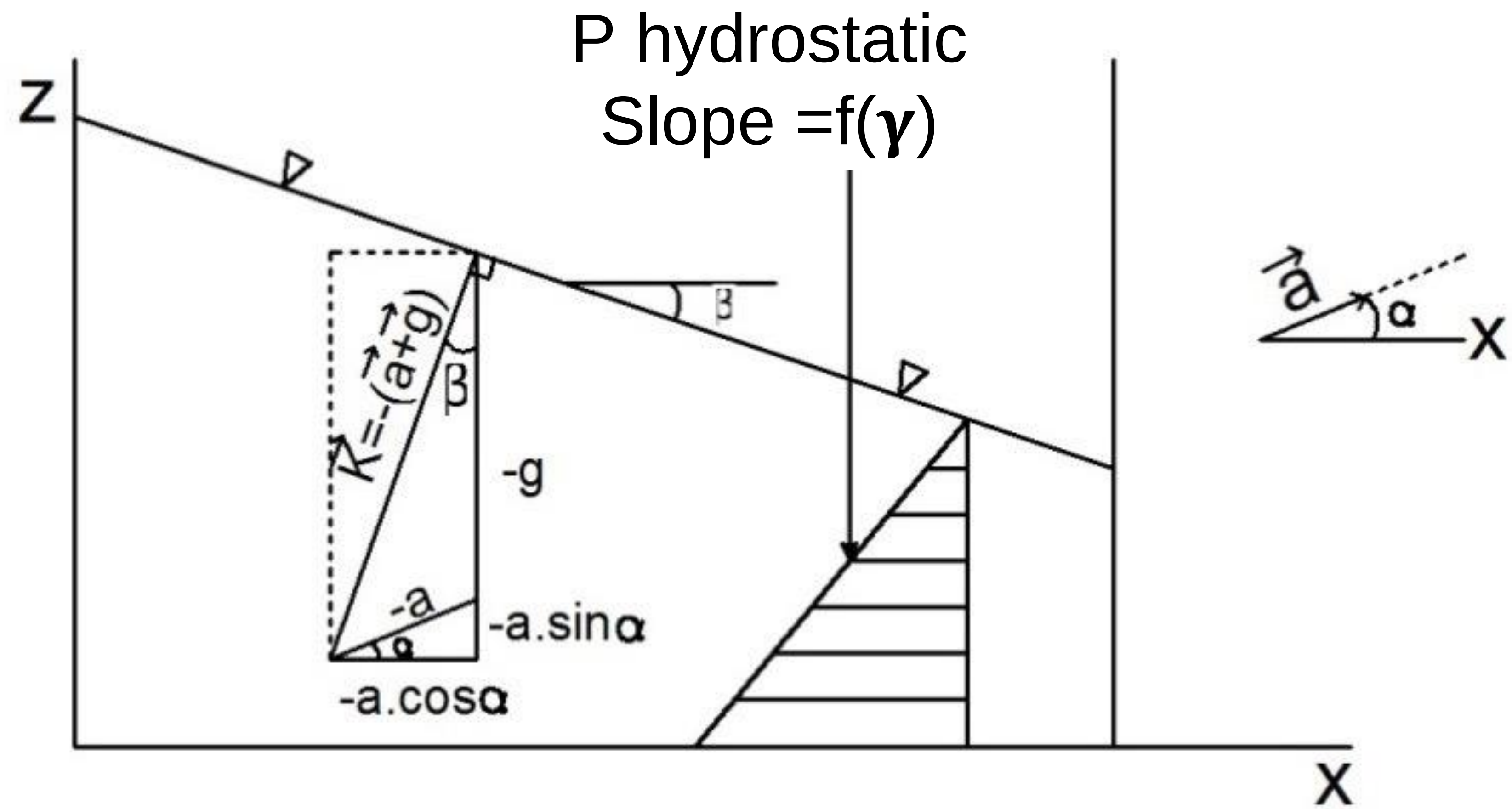

Taking the figure in to consideration, for a fluid having unit mass:

The horizontal volumetric force component is $K_x = -a.\cos\alpha$ ***and***

The vertical volumetric force component is $K_y = -(g + a.\sin\alpha)$

$$dp = -\rho(a.\cos\alpha.dx + (g + a.\sin\alpha)dz)$$

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Considerations:

a. We said that K is perpendicular to the iso-pressure surfaces.

This implies that the free surface of the liquid is perpendicular to the vector ' $(\vec{g} + \vec{a})$ ***'.***

A good example for such a condition is the motion of a body in an elevator. The weight of the body moving in the elevator is a function of both the acceleration of the elevator and the gravitational acceleration.

b. On neo surfaces, we know that dp=0.

$$Z = -\frac{a.\cos\alpha}{g + a.\sin\alpha}.x + C$$

This equation is a linear equation.

c. For x= constant, dx=0

$$\Rightarrow dp = -\rho.(g + a.\sin\alpha)dz$$

$\Rightarrow P = -\gamma' Z + C = \gamma' h + C$ as h and Z are measured in opposite directions.

Accordingly,

- *The pressure distribution along the vertical direction becomes like hydrostatic*
- *The specific weight of the fluid,* γ'*, is seen as if it were distributed*

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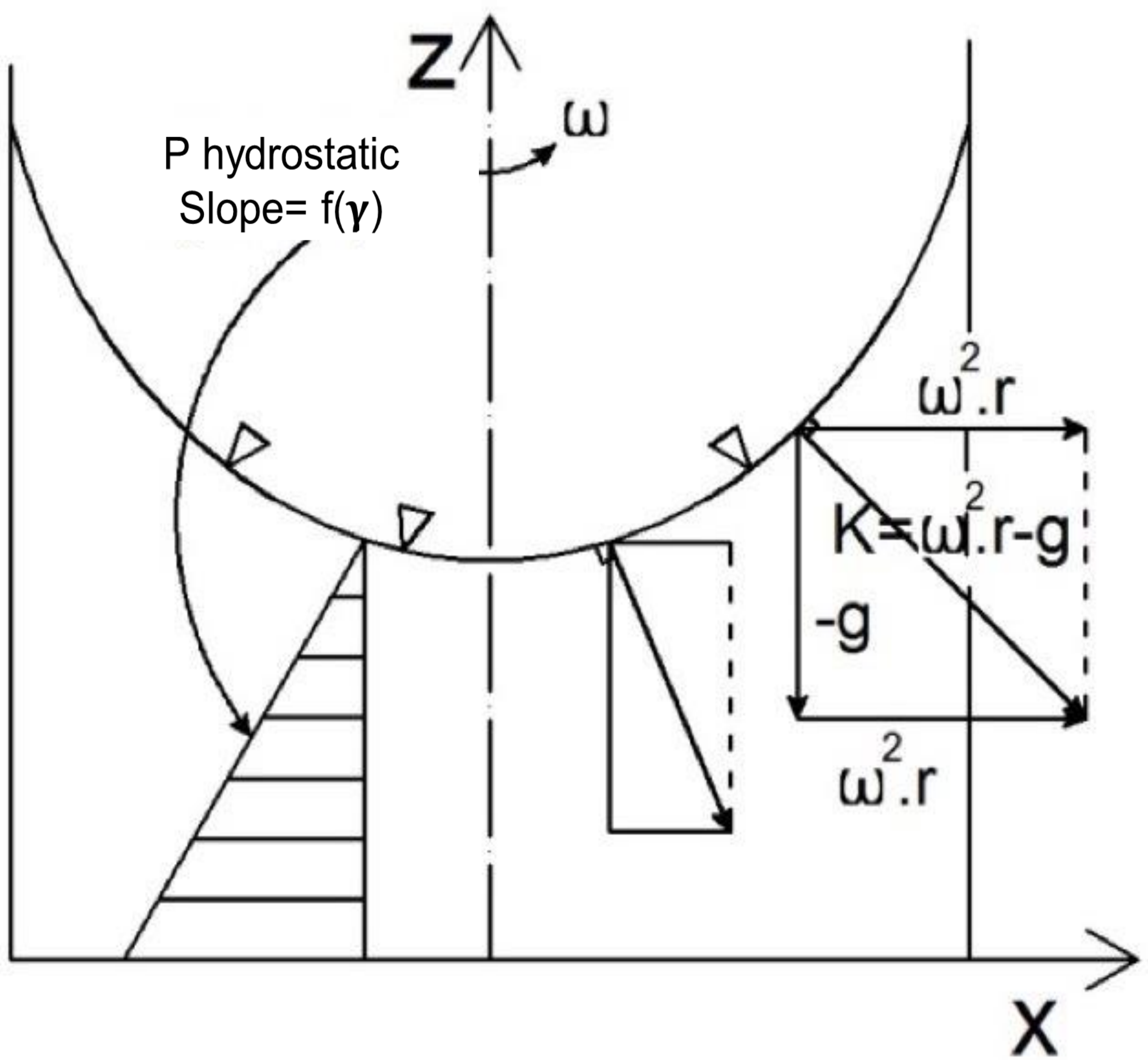
1. A container rotating with a constant angular velocity.

In this case, the free surface of the fluid is not horizontal, but parabolic.

If we take a point on the parabola at a certain distance, r, we will have



$$K_r = \omega^2 . r \, , \; K_z = -g$$

A container rotating with a constant angular velocity

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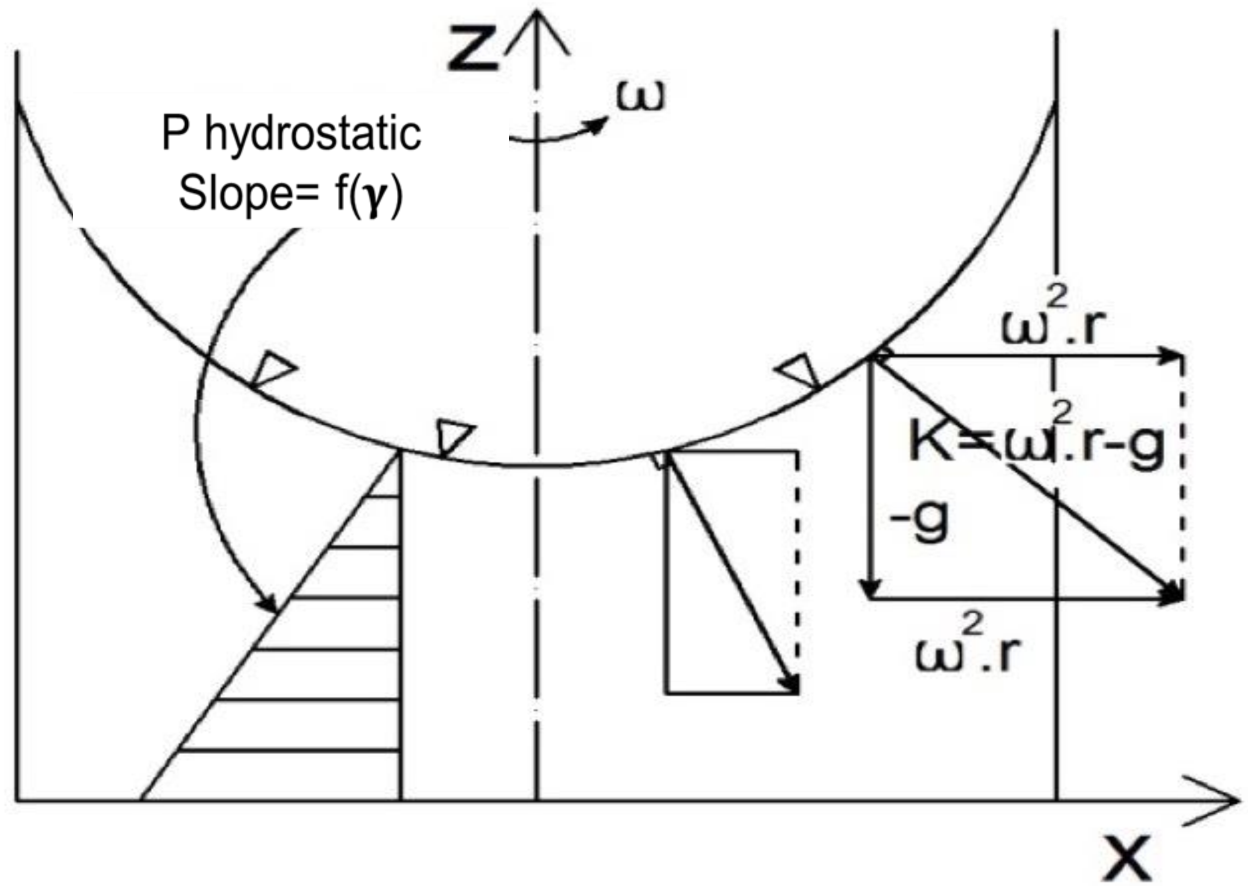

<u>***For a fluid of unit mass, the change in pressure with distance in a certain direction is equal to the specific weight of the body.***</u>

$$\Rightarrow \frac{\partial p}{\partial r} = \rho K_r \text{ and } \frac{\partial p}{\partial z} = -\rho g$$

$$dp = \rho \omega^2 .r.dr - \rho g dz$$

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Considerations:

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a. K is perpendicular to the iso-pressure surface. This implies that, at every place, the free surface of the liquid is perpendicular to the vector ‘ $(\vec{g} + \vec{\omega}^2 .r)$ *’.*

b. In the neo surface, dp=0

$$Z = \frac{\omega^2}{2g} . r^2 + C$$

This is a second order equation and the free surface is a parabola.

c. For r=constant, dr=0.

$$P = -\gamma z + C$$

This is equation is again the hydrostatic pressure equation and C can be determined by considering boundary conditions as done earlier.

- ***This means that, the pressure distribution on a vertical cross section located at a certain distance, r, hydrostatic or***
- ***The pressure at points at of the same depth from the free surface is the same.***


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