Fluid Mechanics

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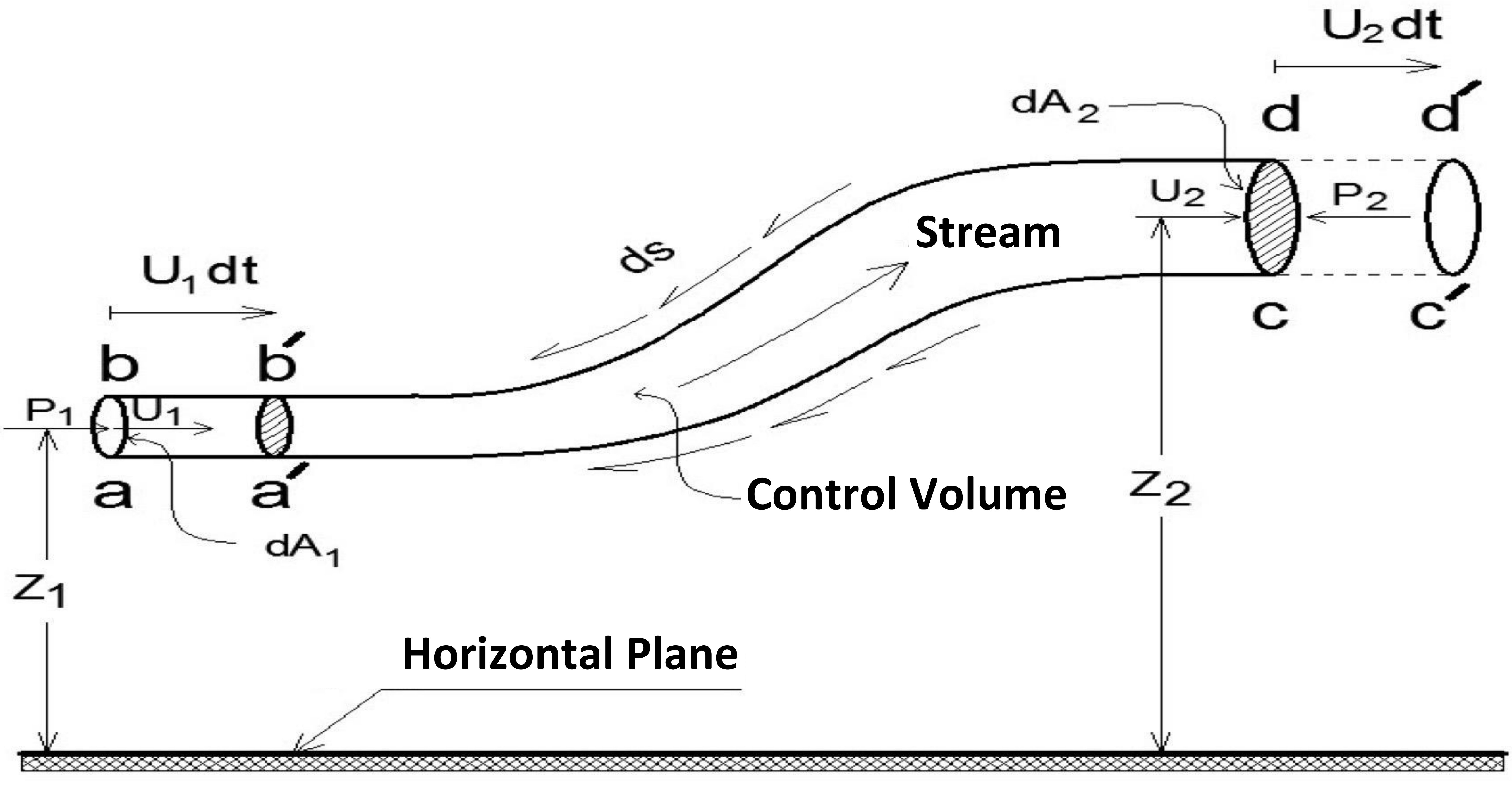
Continuity Equation:

According to the conservation of mass the mass of fluid (m) with in the control volume remains <u>constant</u>.



As the fluid is assumed to be incompressible

$$\rho_1 = \rho_2$$

$$u_1 . dA_1 = u_2 . dA_2$$

This is the general *continuity equation* for incompressible fluids

Energy Equation:

In addition to the above assumptions

*Let's also assume that the work done as a result of the frictional force is **dS***

Based on the law of conservation of energy, the energy of the system at time, t,should be equal to the energy of the system at time, t+dt.

$$E_t = E_{t+dt} + dS$$

The total energy within the system is

$$E_t = EP + ES + EK$$

EP is potential energy ***ES is pressure energy*** ***EK is kinetic energy***

Therefore the total energy at point 1 is

$$EP_1 + ES_1 + EK_1$$

This is the general Energy Equation for *incompressible fluids*

$$u_1.dA_1.z_1 + \frac{p_1}{\gamma}.u_1.dA_1 + u_1.dA_1.\frac{u_1^2}{2g} = u_2.dA_2.z_2 + \frac{p_2}{\gamma}.u_2.dA_2 + u_2.dA_2.\frac{u_2^2}{2g} + dS$$

*From **Continuity equation**, we know that.*

$$u_1.dA_1 = u_2.dA_2$$

Therefore, this equation can be further simplified and will have another form.

So more to come later.

Impulse-Momentum Equation

From physics, we know that the general equation of this is:

$$\vec{F} = \frac{d(m\vec{u})}{dt}$$

If mass (m) is constant

$$\vec{F} = m\left(\frac{d\vec{u}}{dt}\right)$$

$$\vec{F}dt = md\vec{u}$$

The left side of the equation is what is called *Impulse* and that right side of the equation is what is known as *Momentum*

Considering

$$\rho_2 = \rho_1$$

$$\vec{F} = (\rho . u_2 . dA_2)\vec{u}_2 - (\rho . u_1 . dA_1)\vec{u}_1$$

This is the general form of Impulse-Momentum Equation for *incompressible fluids*

From this equation,

$$\vec{F}_x = (\rho . u_2 . dA_2)\vec{u}_{x_2} - (\rho . u_1 . dA_1)\vec{u}_{x_1}$$

We can determine $\vec{F}_y$ and $\vec{F}_z$ **following similar approach**

Finally, F can be calculated as:

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

One dimensional Flow of Ideal Fluids

Let's apply the basic equation developed for incompressible fluid and steady-state flow with in a flow pipe having infinitesimally small cross-sectional area

Ideal fluids → $\tau = 0, \; \mu = 0$

↓

Frictional force is zero

← ***Flow will be uniform at any point in the pipe.***

So in these case :

↓

$$u_1 = v_1 \text{ and } u_2 = v_2$$

FINALLY

$$V_1 . A_1 = V_2 . A_2 = Q$$

$$V_1.A_1 = V_2.A_2 = Q$$

This is the **continuity equation** for *ideal fluid* and the discharge

It implies that

The discharge Q, remains constant at various cross-sections along the flow length

$$(Q_1 = Q_2)$$

Energy equation for 1-D ideal fluids:

Let's bring the general Energy equation we developed earlier

$$u_1.dA_1.z_1 + \frac{p_1}{\gamma}.u_1.dA_1 + u_1.dA_1.\frac{u_1^2}{2g} = u_2.dA_2.z_2 + \frac{p_2}{\gamma}.u_2.dA_2 + u_2.dA_2.\frac{u_2^2}{2g} + dS$$

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g} = H$$

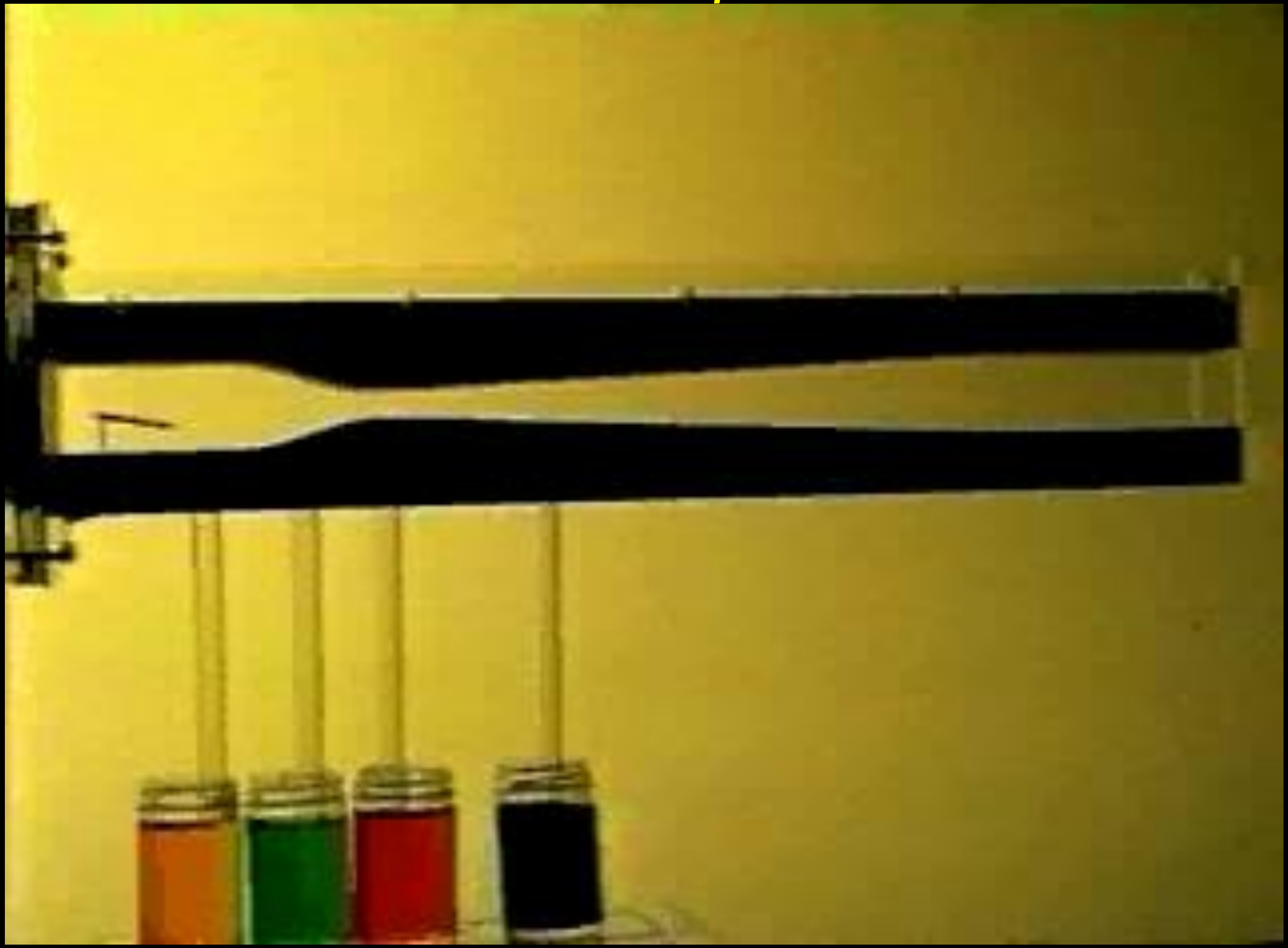
This is the energy equation for ideal fluids

It is called ***Bernoulli's Equation***

As a fluid flows through a converging channel (Venturi channel), the pressure is reduced in accordance with the continuity and Bernoulli equations

As predicted by the Bernoulli equation, an increase in velocity will cause a decrease in pressure

The attached water columns show that the greatest pressure reduction occurs at the narrowest part of the channel



The same principle is used in a garden sprayer so that liquid chemicals can be sucked from the bottle and mixed with water in the hose.

The Impulse-Momentum Equation for Ideal Fluids:

From our previous analysis, we have the following general Impulse-Momentum equation

$$\vec{F} = (\rho . u_2 . dA_2)\vec{u}_2 - (\rho . u_1 . dA_1)\vec{u}_1$$

$$\vec{F} = \rho . Q(\vec{v}_2 - \vec{v}_1)$$

This is the resultant force acting on the control volume.

$$\vec{F}_x = \rho . Q(\vec{v}_{x_2} - \vec{v}_{x_1})$$

Components

$$\vec{F}_x = \rho . Q(\vec{v}_{x_2} - \vec{v}_{x_1})$$

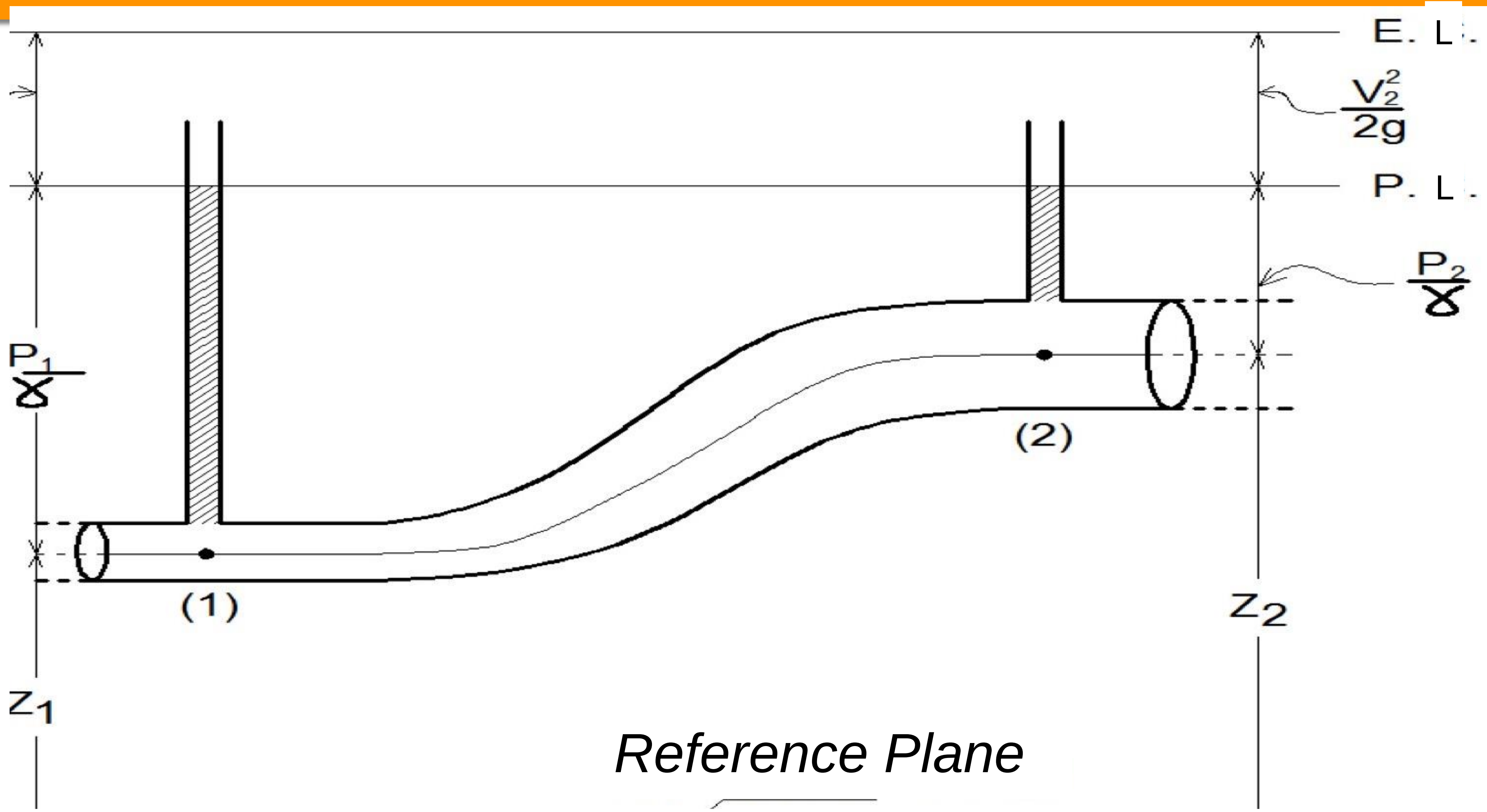
Magnitude of F

$$F = \sqrt{F_x^2 + F_y^2}$$

The physical and geometrical meaning of Bernoulli's equation

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g} = H$$

In ideal fluids, the sum of fluids
__potential energy, pressure energy and kinetic energy__
at various cross-sections remains constant.



We know that:

$$[Z] = m$$

$$\left[\frac{p}{\gamma}\right] = \frac{F/L^2}{F/L^3} = m$$

$$\left[\frac{v^2}{2g}\right] = \frac{L^2/T^2}{L/T^2} = m$$

This implies that H is also in meters.

In its geometric meaning,

$$H$$

is called total hydraulic head

$$Z$$

is called potential head,

$$\frac{p}{\gamma}$$

is called pressure head

$$\frac{v^2}{2g}$$

is called velocity head.

In its physical or mechanical meaning

$$H$$

is called total energy head

$$Z$$

is called potential energy

$$\frac{p}{\gamma}$$

is called pressure energy

$$\frac{v^2}{2g}$$

is called kinetic energy

Part of the equation given as:

$$z + \frac{p}{\gamma}$$

is called Piezometric head

In its physical meaning ***it is called Piezometric energy.***

The geometric location of energy points gives energy line

and

the piezometric heard at various points give peizometric line

In flow of ideal fluids, energy line is always horizontal

To determine the location of piezometric line

We subtract a dimension of $\frac{v^2}{2g}$ **from the energy line**

Depending on the use of absolute or relative pressure in the calculations

the lines are called absolute or relative energy or piezometric lines respectively.

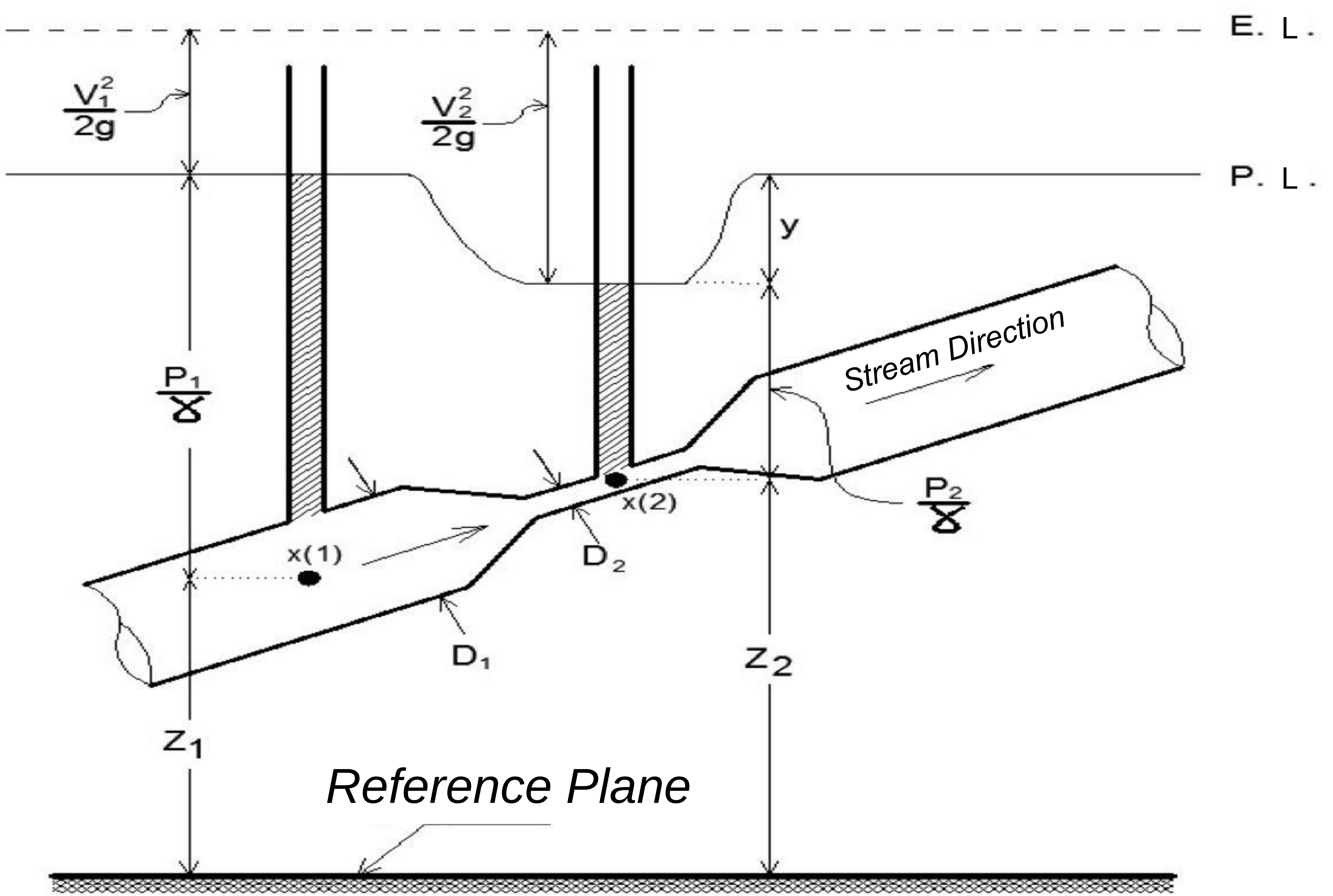
When we are using Bernoulli's equation, the absolute pressure at every point in the flow should be greater or at least equal to absolute evaporative pressure. If it is not given, the absolute evaporative pressure of water is taken as zero.

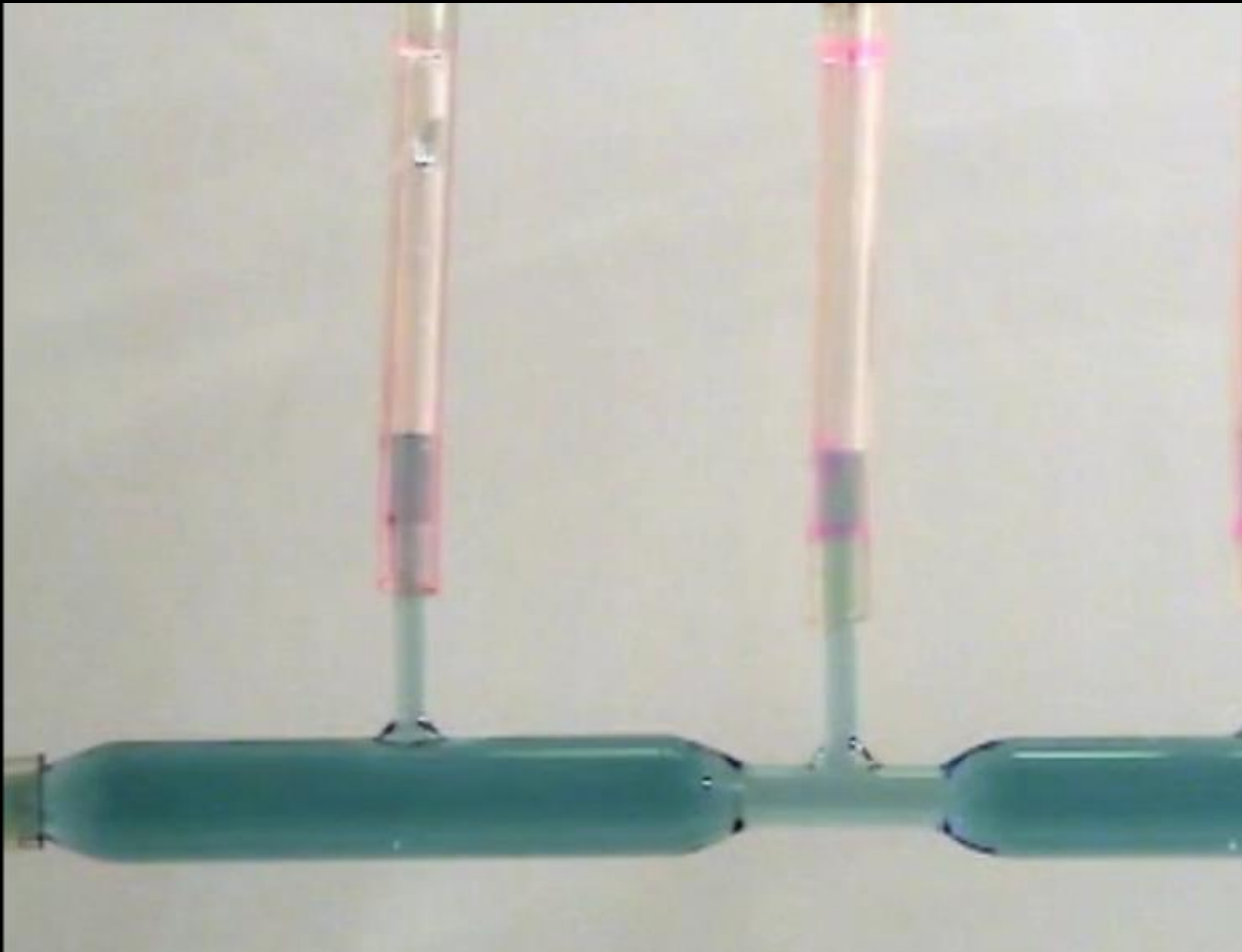
One dimensional flow *in ideal fluids-Applications*

Applications of Bernoulli's Equation

1. Venturi meter

Venture meter is an apparatus used to *measure discharge*





From **continuity equation** we know

$$V_1 . A_1 = V_2 . A_2 = Q$$

$$\frac{V_1}{V_2} = \frac{A_2}{A}$$

*Arranging the terms in the **Bernoulli equation** we get following form*

$$\left(z_1 + \frac{p_1}{\gamma}\right) - \left(z_2 + \frac{p_2}{\gamma}\right) = \left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right)$$

From the figure we can see that

$$\left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right) = y$$

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$$\left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right) = y$$

So →

$$\left(z_1 + \frac{p_1}{\gamma}\right) - \left(z_2 + \frac{p_2}{\gamma}\right) = y$$

We know that discharge, Q, is the product of velocity and cross-sectional area.

$$V_2 = \sqrt{\frac{2g}{1 - \frac{A_1^2}{A_2^2}}}\sqrt{y}$$

$$Q = A_2 . V_2$$

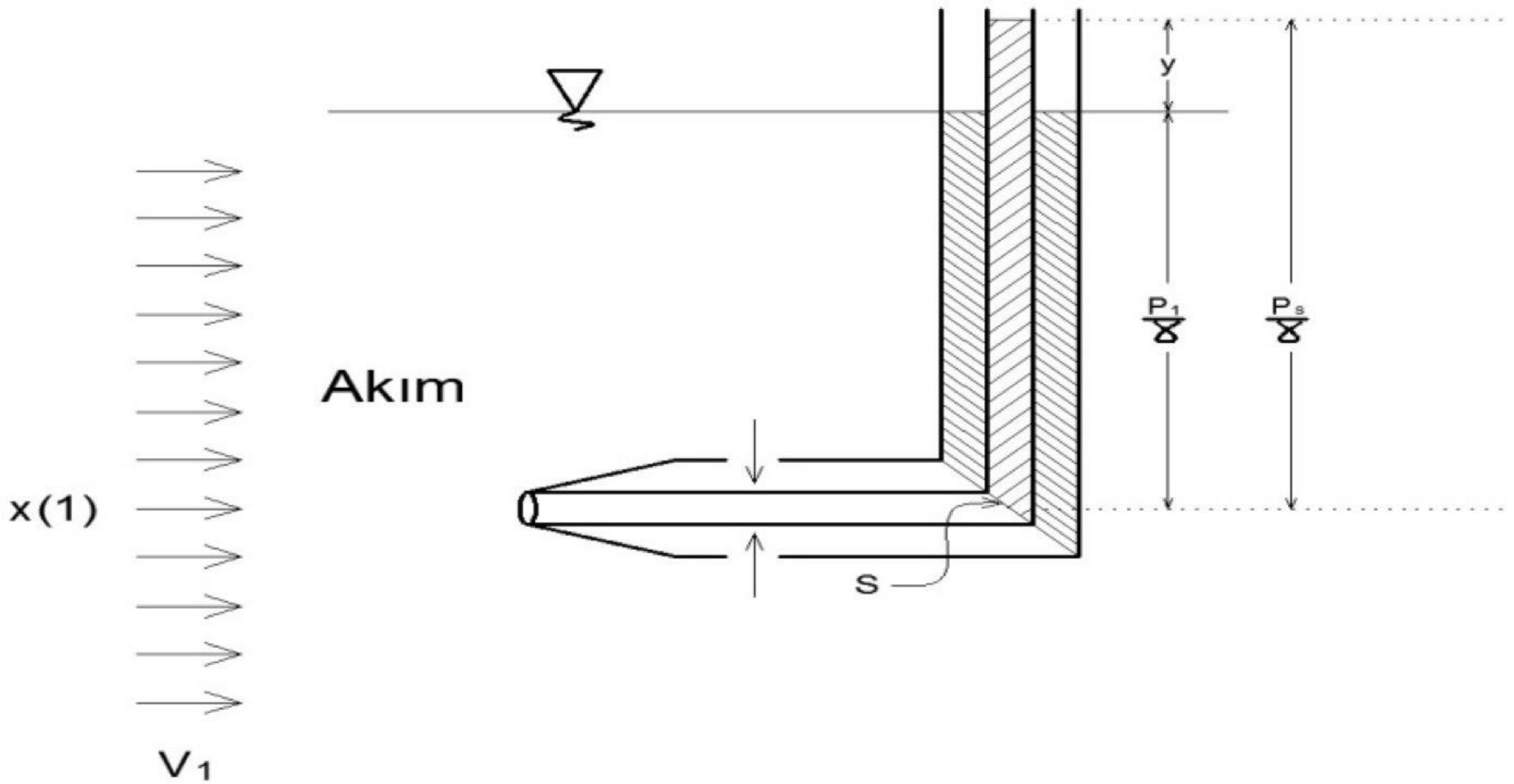
$$Q = A_2 . \sqrt{\frac{2g}{1 - \frac{A_1^2}{A_2^2}}}\sqrt{y}$$

$$Q = C\sqrt{y}$$

It should not be forgotten that *y is the piezometric head or velocity head difference between two points*

2. Pitot Tube

This instrument is used to measure the *velocity of flow*



Writing Bernoulli's equation between points 1 and 2 here again

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g} = H$$

Since

$$z_1 = z_2$$

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

And since the second point is selected on the wall of the tube

$$V_2 = 0$$

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} = \frac{p_2}{\gamma}$$

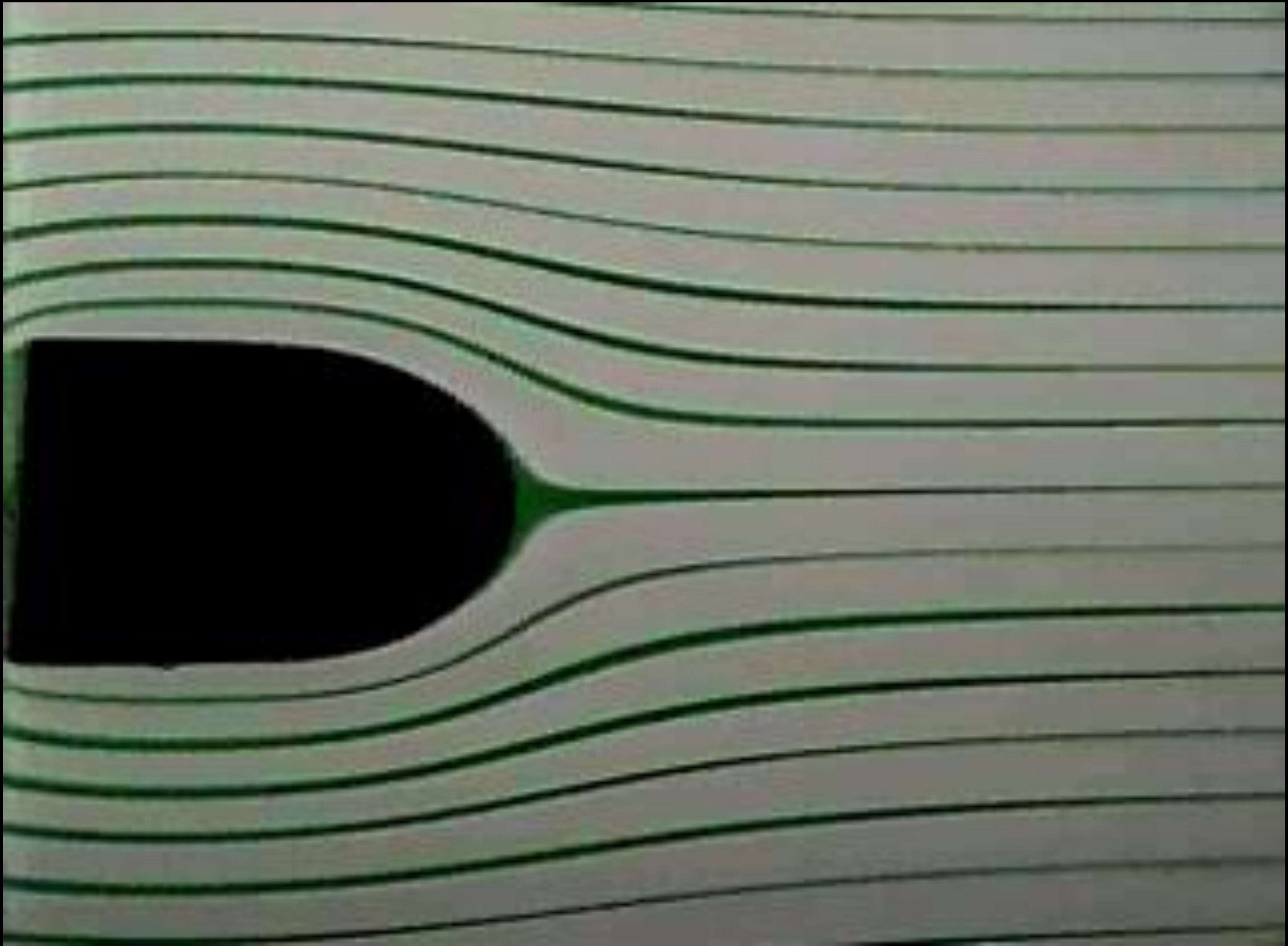
$$P_2 = P_1 + \frac{\rho v_1^2}{2}$$

Right hand side of the above equation is called ***stagnation pressure***. The stagnation pressure is also known as ***Dynamic pressure***.

On any body in a flowing fluid there is a ***stagnation point***

Some of the fluid flows "over" and some "under" the body

The dividing line (the stagnation streamline) terminates at the stagnation point on the body.



From the figure, we can identify that

$$\frac{P_2}{\gamma} - \frac{P_1}{\gamma} = y$$

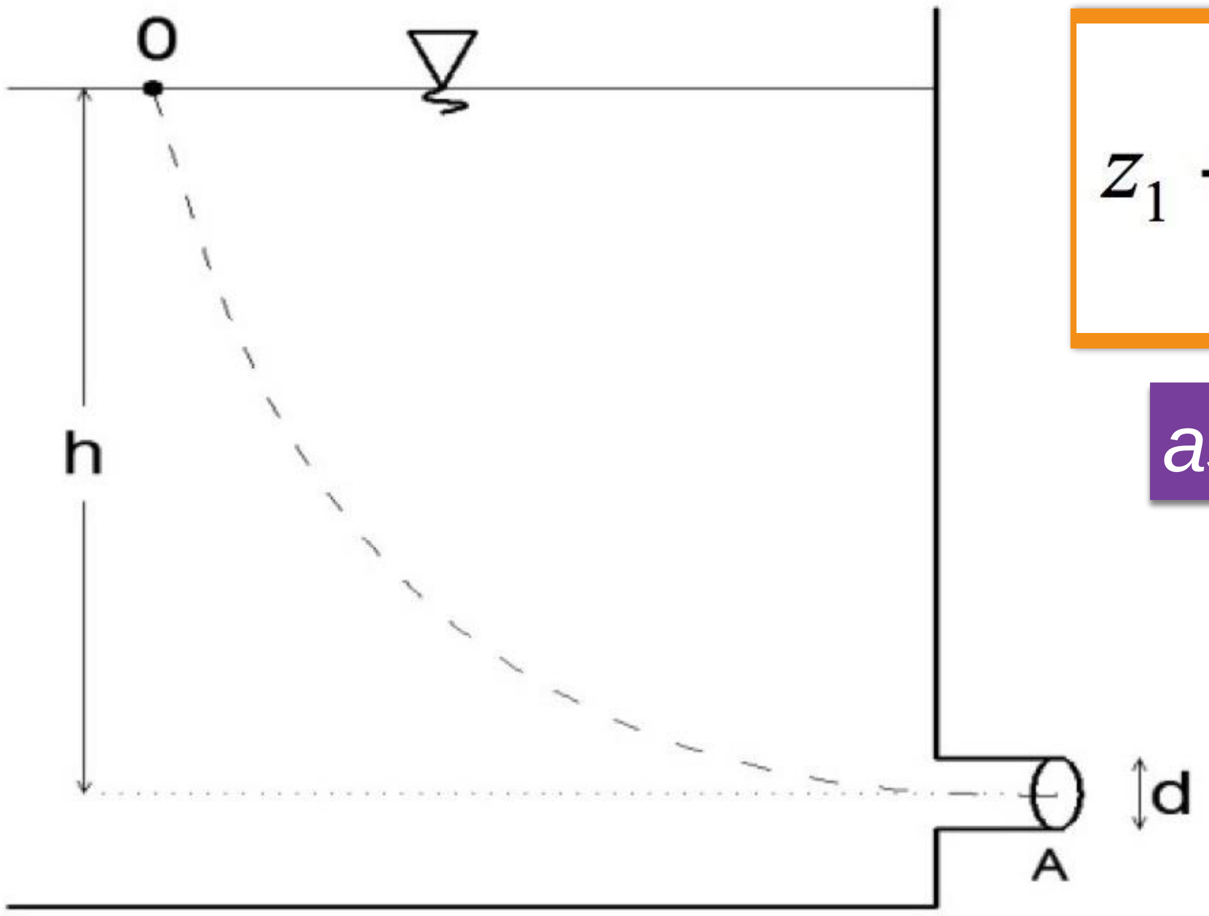
Solving for V, we will get:

$$V = \sqrt{2g.y}$$

Therefore, this equation can be used to determine the velocity of flow at a point of known y.

3. Orifice

If we write Bernoulli's equation between points 1 and 2 indicated in the figure, we will have:



$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g} = H$$

as the datum is taken at the outlet

$$\frac{v_1^2}{2g} = 0$$

$$z_2 = 0$$

$$\frac{p_1}{\gamma} = 0$$

as the point is taken on the free surface and considering relative pressure

$$\frac{p_2}{\gamma} = 0$$

as it is again an open channel flow at point 2

Taking $z_1 = h$ we will have

$$h = \frac{v_2^2}{2g}$$

$$\Rightarrow V_2 = \sqrt{2g.h}$$

In order to calculate discharge, Q:

$$Q = A.V$$

$$\Rightarrow Q = \frac{\pi D^2}{4}\sqrt{2g.h}$$

According to the Bernoulli equation, the velocity of a fluid flowing through a hole in the side of an open tank or reservoir is proportional to the square root of the depth of fluid above the hole.

The greater the depth, the higher the velocity

$$V = \sqrt{2g.y}$$

Similar behavior can be seen as water flows at a very high velocity from the **reservoir behind Glenn Canyon dam in Colorado**

Comments on Energy equation:

What have we done so far ?

In dealing with the energy equation,
we assumed the fluid to be ideal and incompressible

. We then developed the equation in such a way that the terms in the equation are given per unit weight of the fluid

Geometric comments:

Because of this, at points found on the same flow line in a steady-state 1-D, ideal and incompressible fluid flow, the sum of potential (geometric) head, pressure head and velocity head remains constant.

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g} = H$$

Mechanical comments:

Let's think part of the fluid that mg= 1 kg

This implies that

$$m.g.z = z$$

and it is the potential energy

In the same manner

$$m.g\frac{p}{\gamma} = 1\frac{p}{\gamma} = \frac{p}{\gamma}$$

This is pressure energy

$$\frac{1}{2}m.v^2 = \frac{1}{2}.\frac{1}{g}.v^2 = \frac{v^2}{2g}$$

In addition this is the kinetic energy !!!

In this condition,

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g} = H$$

Implies that for ideal, incompressible, steady , 1-D flow, *all points found on the same flow line* , the sum of potential energy, pressure energy and kinetic energy ***remains constant***